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Universes of the Multiverse

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Abstract

The paper's purpose was to investigate the existence of new physical constants. New physical constants for mass, elementary charge, and reduced Planck's constant were found for the hydrogen atom. These new physical constants allowed Schrodinger equations to be solved for new wave functions of the hydrogen atom. These new wave functions of the hydrogen atoms were assumed to be from new universes. Thus, the universes of the multiverse were discovered. The only difference between the universes was the quantum states of the atoms. Particles with much less mass than in our universe were determined to be capable of exceeding our universe's speed of light, all in agreement with Albert Einstein's theory of special relativity. The velocity of a particle in a universe would be limited to below the speed of light in a vacuum of its universe number. The particles and atoms of the universes of the multiverse were shown to be created by a new principle called an atomic particle transition. Simultaneous atomic particle transitions would be shown to transition a quantum system in the quantum state of one universe to a quantum system in the quantum state of another universe. Atomic particle transitions would also be shown to emit extremely energetic photons from hydrogen atoms. These photons would generate an energetic force that would be the basis of a powerful hydrogen photon reaction engine. Gravity in the universes of the multiverse was determined to be an extension of Isaac Newton's universal law of gravitation.

Keywords: Multiverse, parallel dimensions, atomic particle transition

1. Introduction

What if particles could have unlimited discrete size, mass, and density? Then could these particles create atoms that make up the unique unlimited universes of the multiverse? Could there be discrete unlimited universes that simultaneously exist everywhere in three-dimensional space? Could the only difference between the universes of the multiverse be the quantum states of the atoms? Could the unlimited universes of the multiverse be remarkably like our universe? Could the matter and energy of any universe be incompatible with the matter and energy of any other universe? Could the universes of the multiverse be invisible to each other? Could the matter and energy of any universe pass entirely through matter and energy of any other universe as though the other wasn't there? Could wave functions derived from Schrodinger equations validate the existence of the universes of the multiverse? Then could particles of greater numbered universes of the multiverse travel faster than the speed of light in our universe? Might the speed of these particles be limited by the speed of light in a vacuum of the universes of the multiverse? Might a new principle be used to create quantum states of the atoms of the universes called atomic particle transitions? Could this principle be used to develop a future galactic spacecraft that can disappear from a universe by changing its quantum state to that of another universe? Could this future galactic spacecraft travel at velocities way beyond the speed of light in our universe? Would it be possible to create artificial anti-g-forces? Could the flow of time be dependent on the quantum states of the quantum systems of the universes of the multiverse? Could gravity be the result of the multiverse? Could the issues with Isaac Newton's universal law of gravitation be resolved? Could Isaac Newton's universal law of gravitation be validated and extended to the universes of the multiverse? Could this be deduced, all in agreement with Einstein's special theory of relativity? This paper says yes.

The paper's purpose was to investigate the existence of new physical constants. The physical constants, mass, elementary charge, reduced Planck's constant, the speed of light in a vacuum, etc. were found. Motivation to find new values for these physical constants was strong. For if they could be found, they might reveal new atoms, new quantum systems, new electromagnetic waves, and new physical laws.

What is explained in this paper is entirely new. It is related to Albert Einstein's theory of special relativity, Schrodinger's equation, and other classical principles of physics. But the results are entirely new.

Albert Einstein published the theory of special relativity in 1905. This proved that the closer to the speed of light you get a particle, the more massive it becomes, and the more energy is required to achieve its velocity. It proved that particles in our universe can never reach or exceed the speed of light in a vacuum c. Thus, what is deduced here must agree with Albert Einstein's theory of special relativity.

In atomic physics, an atomic electron transition (also called a quantum jump) is a change (or jump) of an electron from one energy level to another within an atom. An atomic particle transition is a similar change (or jump) of an atom between the quantum states of the universes of the multiverse.

Hydrogen atoms are capable of transitioning from the quantum state of our universe to the quantum state of universe number negative one, by atomic particle transitions, making the hydrogen atoms disappear from our universe. This should be immediately testable in the framework of current knowledge. Hydrogen atoms absorbing photons of the exact appropriate electromagnetic wavelength will accomplish this. This wavelength is approximately $9.10x10^{-8}$ m ($3.30x10^{15}$ Hz) (see SECT. V). That this could be done is experimentally distinguishable from existing knowledge.

Multiple universes have been proposed in philosophy, transpersonal psychology, religion, music, astronomy, physics, and all kinds of literature, particularly in science fiction, fantasy, and comic books. Multiple universes have been given many names including "parallel worlds," "parallel universes," "parallel realities," "parallel dimensions," "alternate realities," "alternate timelines," "alternate dimensions," "dimensional planes," and "quantum realities." This is evidence that considerable work has been done on multiple universes over the past 20-30 years. Multiple universes in past scientific literature have been called a "multiverse." A multiverse or multiple universes have been classified as Brian Greene's nine types, Max Tegmark's four levels, M-theory, Cyclic theories, Anthropic principle, Occam's razor, and Modal realism. This paper defines the universes of the multiverse in Euclidean geometry. However, the multiverse does not fit any of these classifications.

This work is an extension of the work of Erwin Schrodinger. Schrodinger equations are solvable in the universes of the multiverse. Schrodinger equations are shown to be capable of determining the discrete wave functions of the universes of the multiverse. This will validate the universes of the multiverse.

The universes of the multiverse have gravity. The properties of the universes will determine the force of gravity in the universes of the multiverse. The organization of the paper is as follows: SECT. I Introduction. SECT. II determines the physical constants required to develop the atoms and physical laws of the universes of the multiverse. SECT. III derives Schrodinger equations for the universes of the multiverse. SECT. IV derives wave functions and properties for the hydrogen atom that shows the universes of the multiverse exist. SUBSECT. A is a mathematical check of the wave functions of the hydrogen atom of the universes of the multiverse. SECT. V defines and discusses the new principle of atomic particle transitions that would not exist if there were not a multiverse. SECT. VI shows physical laws of the universes of the multiverse. SECT. VII defines and discusses electromagnetic waves, photons, a theoretical hydrogen photon engine, and quantum gravity of the universes of the multiverse. SECT. VIII shows the consequences of Albert Einstein's special theory of relativity in the universes of the multiverse. SECT. IX Conclusion.

The Gaussian system of units (Gs) is used for deriving equations rather than the International System of Units (SI). Use of the Gaussian system of units importantly ensures that mechanical and electromagnetic units can be unambiguously derived from the same three base units. This is especially important, in that it is necessary to derive important physical laws of the universes of the multiverse, e.g., Schrodinger equations and their wave functions.

2. Physical Constants of the Universes of the Multiverse

It was hypothesized that we live in a multiverse of discrete unlimited universes that are remarkably like our universe. It was hypothesized that the discrete unlimited universes of the multiverse simultaneously exist everywhere in three-dimensional space. It was hypothesized that the difference between the universes of the multiverse was the quantum state of the atoms. It was hypothesized that the universes of the multiverse would be so similar, that it would be nearly impossible to tell which one you were in. It was hypothesized that the properties of the universes of the multiverse would also be so different that it would be impossible to see any other universe. It was also hypothesized that the universes of the multiverse would be so different that any universe could pass through any other universe as though the other one was not there. It was hypothesized that each universe has its own number k. It was hypothesized that the universes of the multiverse were described by quantum numbers, universe k, principal n, azimuthal l, and magnetic m. It was hypothesized that the universes of the multiverse, electromagnetic waves, photons, and physical constants of the universes of the multiverse were described by the universe quantum numbers. It was hypothesized that if a quantum system changed its universe quantum number it also changed the universe to which it belongs. It was hypothesized that the universe quantum number k was equal to zero for our universe. It was hypothesized that universe quantum numbers k are all integers.

It was hypothesized that quantum numbers, universe k, principal n, azimuthal l, and magnetic m describe values of conserved quantities in the dynamics of a quantum system in the universes of the multiverse. It was hypothesized that quantum numbers can be defined as the sets of numerical values which give acceptable solutions to Schrodinger equations of the hydrogen atom of the universes of the multiverse. SECT. IV shows that quantum numbers, universe k, principal n, azimuthal l, and magnetic m meet this definition of the universe's quantum numbers.

The mass of subatomic particles m, the reduced Planck's constant $\hbar$, the elementary charge e, and the speed of light in a vacuum c are thought to be physical constants whose numerical values never change with time. In this work, it was hypothesized that these and other physical constants whose numerical values never change with time depend on the universe.

It was hypothesized that physical constants, physical properties, and gaussian units took on the following equation format in the universes of the multiverse:

$$\beta_k=\alpha^{\mu k}\,\beta \qquad (1)$$

where k was a universe quantum number that was an integer, β_k were β in the k^{th} universe, α is the fine structure constant, μ was the coefficient of k where μ was an integer or a fraction of integers, and β was a physical constant, physical property, or Gaussian unit in our universe. If two different β had identical gaussian units then the value of μ was the same. If β was dimensionless then μ was zero.

This work was based on the following three hypotheses: First, and most important, was that most everything, e.g., particles, atoms, and electromagnetic waves, photons, quantum systems were discrete scale-models of themselves in the universes of the multiverse. This hypothesis was identical with the equation.

$$\Delta x_k=\alpha^{(-k)}\,\Delta x \qquad (2)$$

where Δx is a displacement in our universe, Δx_k was the same discrete displacement in the k^{th} universe, and μ equaled negative one. Thus, particles, atoms, and electromagnetic waves of the universes of the multiverse would be discrete exact scale-models of each other, but they would vary a lot in discrete size. A quantum system that would change its universe quantum number would modify the discrete displacements of the quantum system.

Second, the velocities were

$$v_k=\alpha^{(-2k)}\,v \qquad (3)$$

where v is a velocity in our universe, v_k was the same velocity in the k^{th} universe, and μ was equal to negative two. A particle that would change its universe quantum number would discretely modify the velocity of the particle.

Third, it was proposed that momentum of mass was conserved in the universes of the multiverse. This yielded the following equations,

$$p_k=m_k\,v_k=\alpha^{ak}\,m\alpha^{(-2k)}\,v=\alpha^{0k}\,p, \qquad (4)$$

where p is momentum in our universe, p_k was same momentum in the k^{th} universe, m is a mass in our universe, m_k was the same discrete mass in the k^{th} universe, μ for momentum p was zero because momentum of mass was conserved between the universes of the multiverse, and μ for mass was a.

The concept of mass-energy equivalence allows mass to be converted to energy and vice versa.

Thus, a particle discretely changes its mass due to this concept. The coefficient μ for a mass equaled two when a was solved for in Eq. (4) so that

$$m_k=\alpha^{2k}\,m \qquad (5)$$

where the rest mass in the universes of the multiverse were m_k and the rest mass in our universe is m.

Electrons would have units of mass; hence the discrete rest masses of electrons in the universes of the multiverse were

$$m_{ek}=\alpha^{2k}\,m_e \qquad (6)$$

where discrete rest masses of electrons in the universes of the multiverse were m_{ek}, and the rest mass of an electron in our universe is m_e .

Protons would have units of mass; hence discrete rest masses of protons in the universes of the

multiverse were

$$m_{pk}=\alpha^{2k}\,m_p \qquad (7)$$

where discrete rest masses of protons in the universes of the multiverse were m_{pk} and the rest mass of a proton in our universe is m_p.

The speed of light in a vacuum in the universes of the multiverse had units of velocity, so that μ equaled negative two the same as Eq. (3). Thus, discrete speeds of light in a vacuum were

$$c_k=\alpha^{(-2k)}\,c \qquad (8)$$

where discrete speeds of light in a vacuum in the universes of the multiverse were c_k and the speed of light in a vacuum in our universe is c. TABLE I shows the speed of light in a vacuum Eq. (8) in the universes of the multiverse c_k .

TABLE I. Calculated approximate values of the speed of light in a vacuum in the universes of the multiverse.

$Universe\ k$	c_k	$Speed\ of\ light\ (m/s)$
4	$\alpha^{-8}c$	3.72×10^{25}
3	$\alpha^{-6}c$	1.98×10^{21}
2	$\alpha^{-4}c$	1.06×10^{17}
1	$\alpha^{-2}c$	5.63×10^{12}
0	$\alpha^{0}c$	3.00×10^{8}
−1	$\alpha^{2}c$	1.60×10^{4}
−2	$\alpha^{4}c$	8.52×10^{-1}
−3	$\alpha^{6}c$	4.54×10^{-5}
−4	$\alpha^{8}c$	2.42×10^{-9}

Note the large values of the speed of light in a vacuum c_k for positive universe numbers in TABLE I. Using the equation for the speed of light in a vacuum Eq. (8) and $c = d_0/\Delta t_0$ it was noted that the length of time for light to travel one light year in the universes of the multiverse was

$$\Delta t_k = \frac{d_0}{c_k} = \frac{d_0}{\alpha^{-2k}c} = \frac{d_0}{\alpha^{-2k}\frac{d_0}{\Delta t_0}} = \frac{\Delta t_0}{\alpha^{-2k}} \tag{9}$$

where Δt_k was the time, it takes for light to travel a light year in universe number k, d_0 is the distance light travels in one light year in our universe, and Δt_0 is the time light takes to travel one light year in our universe. Δt_0 is one year or 525,600 minutes. When k is one, α^{-2k} is approximately 137^2. Thus, the time it took for light of universe number one to travel a light year was approximately twenty-eight minutes. Δt_0 is one year or 31,536,000 seconds. When k is two, α^{-2k} is approximately 137^4. Thus, the time it took for light of universe two to travel a light year was approximately 89.5 milliseconds.

The "classical electron radius" can be found from classical mechanics by equating $E = m_e c^2$ to $E = e^2/r_e$. It is

$$r_e = \frac{e^2}{m_e c^2} \tag{10}$$

where the classical electron radius is r_e and elementary charge is e. This equation was written in the universes of the multiverse as

$$r_{ek} = e_k^2/m_{ek}c_k^2 = \left(\alpha^{jk}e\right)^2/\alpha^{2k}m_e\left(\alpha^{-2k}c\right)^2 = \alpha^{-k}e^2/m_e c^2 = \alpha^{-k}r_e \tag{11}$$

where r_e is the classical electron radius in our universe, r_{ek} was the discrete electron radius in the k^{th} universe, discrete elementary charges of electrons and protons in the universes of the multiverse were e_k , the elementary charge in our universe is e, μ for r_e was negative one because r_e had units of a displacement, and μ for the elementary charge was j.

Discrete elementary charges of electrons and protons in the universes of the multiverse were

$$e_k = \alpha^{-\frac{3}{2}k}e \tag{12}$$

where μ for e was equal to j that was equal to negative three halves when j was solved for in Eq. (11).

The Bohr radius is

$$a_0 = \hbar^2/m_e e^2 \tag{13}$$

where a_0 is the Bohr radius and $\hbar$ is the reduced Planck's constant. This same equation in the universes of the multiverse was

$$a_k = \hbar_k^2/m_{ek}e_k^2 = \left(\alpha^{bk}\hbar\right)^2/\alpha^{2k}m_e\left(\alpha^{-\frac{3}{2}k}e\right)^2 = \alpha^{-k}\hbar^2/m_e e^2 = \alpha^{-k}a_0 \tag{14}$$

where a_k were the discrete Bohr radii in the universes of the multiverse, discrete reduced Planck's constants in the universes of the multiverse were $\hbar_k$, μ for a_0 was negative one because a_0 had units of a displacement, and μ for the reduced Planck's constant was b.

Then, reduced Planck's constants in the universes of the multiverse were

$$\hbar_k = \alpha^{-k}\hbar \tag{15}$$

where μ for $\hbar$ was equal to b that was equal to negative one when b was solved for in Eq. (14). From Eq. (15) Planck's constants in the universes of the multiverse were

$$h_k = \alpha^{-k}h \tag{16}$$

where discrete Planck's constants in the universes of the multiverse were h_k and Planck's constant in our universe is h.

The fine structure constant α is

$$\alpha = e^2/\hbar c. \tag{17}$$

The fine structure constants in the universes of the multiverse were

$$\alpha_k = \frac{e_k^2}{\hbar_k c_k} = \frac{\left(\alpha^{-\frac{3}{2}k}e\right)^2}{\alpha^{-k}\hbar\alpha^{-2k}c} = \alpha^{fk}e^2/\hbar c = \alpha^{fk}\alpha \tag{18}$$

where α_k were the fine structure constants in the universes of the multiverse and μ for α was equal to f that was equal to zero when f was solved for in Eq. (18). In other words, the dimensionless fine structure constant α would be of the same value in all the universes of the multiverse.

3. Schrodinger Equations for the Universes of the Multiverse

It was hypothesized that immutable laws of physics, including the Schrodinger equation, applied to the universes of the multiverse. Laws of physics for the universes of the multiverse were found by replacing the classical physical constants in physical laws of our universe with the appropriate physical constants of the other universes. In this manner the nonrelativistic Schrodinger equations for the universes of the multiverse were found by replacing the classical physical constants in the Schrodinger equation with physical constants of the other universes of the multiverse. This method was used throughout the paper to derive laws of physics that applied to the universes of the multiverse. It should be noted that when this was done for immutable physical laws the results were consistent.

The time-dependent Schrodinger equation [1] is usually given as a postulate of quantum theory.

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = \hat{H}\psi(r,t) \tag{19}$$

where the imaginary unit is i, a partial derivative with respect to time is indicated by $\partial/\partial t$, the Hamiltonian is $\hat{H}$, the wave function is $\psi(r, t)$, the radius of a wave function is r, and time is t.

This time-dependent Schrodinger equation was modified for the universes of the multiverse by replacing $\hbar$, $\psi(r, t)$, $\hat{H}$ with $\hbar_k$, $\psi_k(r, t)$, $\hat{H}_k$. Thus,

$$i\hbar_k \frac{\partial}{\partial t}\psi_k(r,t) = \hat{H}_k \psi_k(r,t) \tag{20}$$

where reduced Planck's constants in the universes of the multiverse were $\hbar_k$, wave functions in the universes of the multiverse were ψ_k (r, t), and Hamiltonians in the universes of the multiverse were $\hat{H}_k$. It would be shown that these Schrodinger equations are entirely general and can be solved for the quantum states of quantum systems of the universes of the multiverse.

The preceding equations were expanded into three-dimensional time-dependent Schrodinger equations of the universes of the multiverse for the hydrogen atom. These Schrodinger equations for the hydrogen atom assumed that the protons were at fixed positions (the Born-Oppenheimer approximation [2]).

$$i\hbar_k \frac{\partial}{\partial t}\psi_k(r,t) = -\frac{\hbar_k^2}{2m_{ek}}\nabla^2\psi_k(r,t) + V_k(r,t)\psi_k(r,t) \tag{21}$$

where the Laplacian is ∇^2, rest masses of electrons m_{ek} in the universes of the multiverse were Eq. (6), and electrostatic potential energies of the hydrogen atom in the universes of the multiverse were $V_K(r, t)$. They described how the universe's quantum states evolved over time.

Physical constants Eqs. (6), (15) were inserted into Eq. (21) to put it in terms of physical constants of our universe that gave

$$i\alpha^{-k}\hbar \frac{\partial}{\partial t}\psi_k(r,t) = -\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2\psi_k(r,t) + V_k(r,t)\psi_k(r,t). \tag{22}$$

These were three-dimensional time-dependent Schrodinger equations for the universes of the multiverse for the hydrogen atom in terms of classical physical constants.

Now ψ_k (r, t) was written as the product of ψ_k (r) and f_k (t). Eq. (22) was written as

$$\psi_k(r) i\alpha^{-k}\hbar \frac{df_k(t)}{dt} = f_k(t)\left[-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2 + V_k(r)\right]\psi_k(r) \tag{23}$$

where the functions of time in the universes of the multiverse were $f_k(t)$, time-independent wave functions in the universes of the multiverse were $\psi_k(r)$, and the time-independent electrostatic potential energies of the hydrogen atom in the universes of the multiverse were V_k (r).

Then variables were separated that gave

$$\frac{i\alpha^{-k}\hbar}{f_k(t)}\frac{df_k(t)}{dt} = \frac{1}{\psi_k(r)}\left[-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2 + V_k(r)\right]\psi_k(r) \tag{24}$$

The left-hand sides of Eq. (23) were only a function of time, and the right-hand sides were only a function of r. This was because both sides were equal to the same constants. Because the right-hand sides of Eq. (24) had the units of energy, the constants (that were total energy eigenvalues) were designated E_k .

The right-hand sides of Eq. (24) now became ordinary differential equations.

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2\psi_k(r) + V_k(r)\psi_k(r) = E_k\psi_k(r) \tag{25}$$

where the total energy eigenvalues in the universes of the multiverse were E_k. The latter equations were three-dimensional time-independent Schrodinger equations of the universes of the multiverse, of the hydrogen atom in terms of classical physical constants.

4. Wave Functions and Properties of the Hydrogen Atom of the Universes of the Multiverse

This section begins with the equation that ended the previous section. Time-independent Schrodinger Eq. (25) for the hydrogen atom for the universes of the multiverse were converted to polar coordinates.

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_k(r)}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_k(r)}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi_k(r)}{\partial\phi^2}\right] + V_k(r)\psi_k(r) = E_k\psi_k(r) \tag{26}$$

where the polar coordinates were r, θ, $\emptyset$.

Electrostatic potential energies for the hydrogen atom in the universes of the multiverse were found by inserting elementary charges Eq. (12) into the formula for potential energies.

$$V_k(r) = -\frac{e_k^2}{r} = -\frac{\left(\alpha^{-\frac{3}{2}k}e\right)^2}{r} = -\alpha^{-3k}\frac{e^2}{r} \tag{27}$$

Then Eq. (27) were substituted into Eq. (26) that gave

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_k(r)}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_k(r)}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi_k(r)}{\partial\phi^2}\right] - \alpha^{-3k}\frac{e^2}{r}\psi_k(r) = E_k\psi_k(r). \tag{28}$$

These were nonrelativistic time-independent Schrodinger equations for the hydrogen atom in the universes of the multiverse, in terms of classical physical constants, that were solved to find the radial wave functions and spherical harmonics of the hydrogen atom in the universes of the multiverse.

Wave functions ψ_k (r) were factored into $R(r)_{(k)nl}$ $Y_l^m(\theta,\emptyset)$, where $R(r)_{(k)nl}$ were radial wave functions in the universes of the multiverse and $Y_l^m(\theta,\emptyset)$ were spherical harmonics.

Radial wave functions $R(r)_{(k)nl}$ then obeyed

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{dR(r)_{(k)nl}}{dr}\right) + l(l+1)R(r)_{(k)nl}\right] - \alpha^{-3k}\frac{e^2}{r}R(r)_{(k)nl} = E_kR(r)_{(k)nl} \tag{29}$$

The latter equations were radial type of equations for the hydrogen atom for the universes of the multiverse in terms of classical physical constants.

The latter equations were solved for the radial wave functions $R(r)_{(k)nl}$. Thus, we have

$$R(r)_{(k)nl} = \left(\frac{2}{na_k}\right)^{\frac{3}{2}}\left(\frac{(n-l-1)!}{2n[(n+l)!]^3}\right)^{\frac{1}{2}} e^{-\frac{r}{na_k}}\left(\frac{2r}{na_k}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right) \tag{30}$$

where the radial wave functions of the hydrogen atom in the universes of the multiverse were $R(r)_{(k)nl}$ and the Bohr radii in the universes of the multiverse were a_k Eq. (14). Eq. (30) were the radial wave functions of the hydrogen atom in the universes of the multiverse. The radial wave functions $R(r)_{(k)nl}$ were checked later in Subsection A.

The associated Laguerre polynomials were obtained according to the following formula:

$$L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right) = \sum_{i=0}^{n-l-1} \frac{(-1)^i[(n+l)!]^2\left(\frac{2r}{na_k}\right)^i}{i!(n-l-1-i)!(2l+1+i)!}. \quad (31)$$

The total energy eigenvalue E of the hydrogen atom is

$$E = -\frac{m_e e^4}{2\hbar^2 n^2} \quad (32)$$

Discrete total energy eigenvalues E_k of the hydrogen atom in the universes of the multiverse from Eqs. (6), (12), (15) were

$$E_k = -\frac{m_{ek} e_k^4}{2\hbar_k^2 n^2} = -\frac{(\alpha^{2k} m_e)\left(\alpha^{-\frac{3}{2}k} e\right)^4}{2(\alpha^{-k}\hbar)^2 n^2} = -\frac{\alpha^{-2k} m_e e^4}{2\hbar^2 n^2} \quad (33)$$

where the discrete total energy eigenvalues were E_k . The term e^4 was replaced in Eq. (33) by using $e^4 = \alpha^2\hbar^2c^2$ derived from the fine structure constant Eq. (17) that gave

$$E_k = -\frac{\alpha^{-2k} m_e \alpha^2 \hbar^2 c^2}{2\hbar^2 n^2} = -\alpha^{2-2k}\frac{m_e c^2}{2n^2} \quad (34)$$

Energy eigenvalues E_k were allowed energy levels of the of the hydrogen atom in the universes of the multiverse. Eq. (30) and Eq. (34) were mathematically checked in Eq. (29) in Subsect. A. TABLE II shows approximate values of allowed energy levels for the ground state of the hydrogen atom in the universes of the multiverse Eq. (34).

TABLE II. Calculated approximate values of allowed energy levels for the ground state of the hydrogen atom in the universes of the multiverse.

$Universe\ k$	E_k	$Energy\ level\ (eV)$
4	$-\alpha^{-6}\frac{m_e c^2}{2}$	-1.69×10^{18}
3	$-\alpha^{-4}\frac{m_e c^2}{2}$	-8.99×10^{13}
2	$-\alpha^{-2}\frac{m_e c^2}{2}$	-4.79×10^{9}
1	$-\frac{m_e c^2}{2}$	-2.55×10^{5}
0	$-\alpha^{2}\frac{m_e c^2}{2}$	-13.6
-1	$-\alpha^{4}\frac{m_e c^2}{2}$	-7.25×10^{-4}
-2	$-\alpha^{6}\frac{m_e c^2}{2}$	-3.86×10^{-8}
-3	$-\alpha^{8}\frac{m_e c^2}{2}$	-2.06×10^{-12}
-4	$-\alpha^{10}\frac{m_e c^2}{2}$	-1.10×10^{-16}

Note the large magnitude energy levels for the ground state of the hydrogen atom in the positive universes of the multiverse.

The left-hand sides of Eq. (23) derive ordinary differential equations for the hydrogen atom in the universes of the multiverse where the independent variable was time. This gave

$$\frac{i\alpha^{-k}\hbar}{f_k(t)}\frac{df_k(t)}{dt} = E_k \quad (35)$$

This is the same as

$$\frac{1}{f_k(t)}\frac{df_k(t)}{dt} = -i\alpha^k\frac{E_k}{\hbar} \quad (36)$$

These equations were solved to yield

$$f_k(t) = e^{-i\alpha^k\frac{E_k}{\hbar}t} \quad (37)$$

Substituting Eq. (34) into Eq. (37) gave $f_k(t)$ for the hydrogen atom.

$$f_k(t) = e^{-i\alpha^k\frac{-\alpha^{2-2k}\frac{m_e c^2}{2n^2}}{\hbar}t} = e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t} \quad (38)$$

From above the wave functions $\psi_k(r)$ were the product of $R(r)_{(k)nl}$ Eq. (30) and $Y_l^m(\theta,\emptyset)$. From SECT. III [See the text right before Eq. (23).] the wave functions of the hydrogen atoms in the universes of the multiverse $\psi_k(r, t)$ were the product of $\psi_k(r)$ and $f_k(t)$. $f_k(t)$ was Eq. (38). Thus, $\psi_k(r, t)$ in terms of its three factors were

$$\psi_k(r,t) = \left(\frac{2}{na_k}\right)^{\frac{3}{2}}\left(\frac{(n-l-1)!}{2n[(n+l)!]^3}\right)^{\frac{1}{2}} e^{-\frac{r}{na_k}}\left(\frac{2r}{na_k}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right) Y_l^m(\theta,\phi) e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t} \quad (39)$$

Eq. (39) were the wave functions of the hydrogen atom in the universes of the multiverse $\psi_k(r,t)$. Thus, universe k, principal n, azimuthal l, and magnetic m quantum numbers gave acceptable solutions to Schrodinger equations for the hydrogen atom in the universes of the multiverse. And the quantum numbers of the universes of the multiverse met the same definition as the quantum numbers of our universe.

Thus, the unlimited universes of the multiverse would be differentiated by wave functions or quantum states. And the quantum state of matter or energy determines which universe of the multiverse it would be in. The position or location of the matter or energy in three-dimensional space would be irrelevant.

Conservation of energy says the energy of a particle is equal to its kinetic energy plus its potential energy. In other words, the kinetic energy plus the electrostatic potential energy of the electron of the hydrogen atom is equal to its total energy eigenvalue where the potential energy and total energy eigenvalue are negative. And total energy eigenvalues are equal to one half potential energies for the hydrogen atom. This meant that the total energy eigenvalues except for sign were equal to the rotational kinetic energies of the electron of the hydrogen atom in the universes of the multiverse. This was true whether the electron orbital was circular or not (consequence of the virial theorem [3]). Thus, it was for the hydrogen atom that the absolute values of total energy eigenvalues E_k were equal to rotational kinetic energies of the electron in the universes of the multiverse.

$$|E_k| = \frac{1}{2} m_{ek} {v'_k}^2 \quad (40)$$

where $|E_k|$ were the absolute value of discrete total energy eigenvalues of the hydrogen atom of the universes of the multiverse and discrete tangential velocities of the electron of the hydrogen atom in the universes of the multiverse were v'_k. Solving for v'_k gave

$$v'_k = \sqrt{\frac{2|E_k|}{m_{ek}}} = \alpha^{-k} \sqrt{\frac{2|E_k|}{m_e}} \quad (41)$$

Substituting total energy eigenvalues Eq. (34) into Eq. (41) gave

$$v'_k = \alpha^{-k} \sqrt{\frac{2\left(\alpha^{2-2k} \frac{m_e c^2}{2n^2}\right)}{m_e}} = \alpha^{-k} \sqrt{\alpha^{2-2k} \frac{c^2}{n^2}} = \alpha^{1-2k} \frac{c}{n} \quad (42)$$

The ratio of the discrete tangential velocities of the electron of the hydrogen atom Eq. (42) to the discrete speeds of light in a vacuum Eq. (8) in the universes of the multiverse were

$$\frac{v'_k}{c_k} = \frac{\alpha^{1-2k} c}{\alpha^{-2k} c n} = \frac{\alpha}{n} \quad (43)$$

Thus, the ratio of the tangential velocities of the electron of the hydrogen atom to the speeds of light in a vacuum were α divided by the principal quantum number n for all quantum numbers in all the universes of the multiverse.

From Eq. (38) discrete angular frequencies of the wave function of the hydrogen atom in the universes of the multiverse were

$$\omega_k = \alpha^{2-k} \frac{m_e c^2}{2\hbar n^2} \quad (44)$$

where the discrete angular frequencies of the wave function of the hydrogen atom in the universes of the multiverse were ω_k. And discrete ordinary frequencies, from Eq. (44), of the wave function of the hydrogen atom in the universes of the multiverse, were

$$f_k = \alpha^{2-k} \frac{m_e c^2}{2\pi 2\hbar n^2} = \alpha^{2-k} \frac{m_e c^2}{2hn^2} \quad (45)$$

where the discrete ordinary frequencies of the wave function of the hydrogen atom in the universes of the multiverse were f_k.

The de Broglie wavelength is $\lambda' = h/p$. The de Broglie wavelengths of the electron of the hydrogen atom in the universes of the multiverse were

$$\lambda'_k = \frac{h_k}{p_k} = \frac{h_k}{m_{ek} v'_k} \quad (46)$$

where the de Broglie wavelengths of the electron of the hydrogen atom in the universes of the multiverse were λ'_k and the momentum of the electron of the hydrogen atom in the universes of the multiverse was p_k. Then m_{ek} was replaced by $\alpha^k \hbar^2 / a_k e^2$ derived from Eq. (14), h_k was replaced by Eq. (16), and v'_k was replaced by Eq. (42) in Eq. (46) that gave

$$\lambda'_k = \frac{\alpha^{-k} h}{\alpha^k \frac{\hbar^2}{a_k e^2} \alpha^{1-2k} \frac{c}{n}} = \frac{h}{\frac{\hbar^2}{a_k e^2} \alpha \frac{c}{n}} = \frac{2\pi a_k n e^2}{\alpha \hbar c} \quad (47)$$

Then, substituting $e^2 = \alpha\hbar c$ dderived from Eq. (17), the de Broglie wavelengths of the electron of the hydrogen atom in the universes of the multiverse λ'_k were

$$\lambda'_k = 2\pi a_k n \quad (48)$$

It was hypothesized that the electron of the hydrogen atom in the universes of the multiverse followed a circular orbit. So, the discrete radii of the electron of the hydrogen atom in the universes of the multiverse were

$$r'_k = a_k n \quad (49)$$

where the discrete radii of the electron of the hydrogen atom in the universes of the multiverse were r'_k. Then, it was hypothesized that the wave function of the hydrogen atom in the universes of the multiverse also followed a circular orbit. And it was hypothesized that discrete radii of the wave function of the hydrogen atom in the universes of the multiverse were the same as the radii of the electron orbit Eq. (49). Thus,

$$r_k = r'_k = a_k n \quad (50)$$

where the discrete radii of the wave function of the hydrogen atom in the universes of the multiverse were r_k.

Then the discrete tangential velocities of the electron of the hydrogen atom in the universes of the multiverse were

$$v'_k = \frac{2\pi r'_k}{t'_k} \quad (51)$$

where the discrete tangential velocities of the electron of the hydrogen atom in the universes of the multiverse were v'_k, and the time (period) for one rotation of the electron orbit of the hydrogen atom in the universes of the multiverse were t'_k. Then, because t'_k were equal to one divided by the discrete ordinary frequencies of the electron orbit of the hydrogen atom in the universes of the multiverse,

$$v'_k = 2\pi r'_k f'_k \quad (52)$$

where the discrete ordinary frequencies of the electron orbit of the hydrogen atom in the universes of the multiverse were f'_k. Then,

$$f'_k = \frac{v'_k}{2\pi r'_k} \quad (53)$$

Substituting Eq. (42) and Eq. (49) into Eq. (53) the discrete ordinary frequencies of the electron orbit would be

$$f'_k = \frac{\alpha^{1-2k} c}{2\pi a_k n^2} \quad (54)$$

Now substituting Eq. (14) into Eq. (54) and $e^2 = \alpha\hbar c$ from Eq. (17) into Eq. (14) the discrete ordinary frequencies of the electron orbit became

$$f'_k = \frac{\alpha^{1-2k} c m_e e^2}{2\pi \alpha^{-k} \hbar^2 n^2} = \frac{\alpha^{1-2k} c m_e \alpha \hbar c}{2\pi \alpha^{-k} \hbar^2 n^2} = \alpha^{2-k} \frac{m_e c^2}{hn^2} \quad (55)$$

Thus, the discrete ordinary frequencies of the wave function of the hydrogen atom in the universes of the multiverse Eq. (45) were half the discrete ordinary frequencies of the electron orbit of the hydrogen atom in the universes of the multiverse Eq. (55).

The discrete tangential velocities of the wave function of the hydrogen atom in the universes of the multiverse were

$$v_k = \frac{2\pi r_k}{t_k} = 2\pi a_k n f_k \tag{56}$$

where the tangential velocities of the wave function of the hydrogen atom of the universes of the multiverse were v_k, the time (period) for one rotation of the wave function of the hydrogen atom in the universes of the multiverse were t_k, and t_k were equal to one divided by the discrete ordinary frequencies f_k of the wave function orbit of the hydrogen atom in the universes of the multiverse. When Eq. (14) and Eq. (45) were substituted into Eq. (56) the latter equations became

$$v_k = \frac{2\pi\alpha^{-k}\hbar^2 n}{m_e e^2}\frac{\alpha^{2-k}m_e c^2}{2hn^2} = \frac{\alpha^{2-2k}\hbar c^2}{e^2 n} \tag{57}$$

When $e^2 = \alpha\hbar c$ from Eq. (17) was substituted into Eq. (57) the discrete velocities of the wave function of the hydrogen atom of the universes of the multiverse v_k became

$$v_k = \frac{\alpha^{2-2k}\hbar c^2}{\alpha\hbar c n} = \alpha^{1-2k}\frac{c}{n} \tag{58}$$

Thus, the discrete velocities of the wave function of the hydrogen atom Eq. (58) were the same as the discrete velocities of the electron of the hydrogen atom Eq. (42) in the universes of the multiverse.

The wave functions Eq. (39) and properties of the hydrogen atom in the universes of the multiverse were shown to be real. Thus, wave functions or the universe's quantum states of all atoms of the chemical elements would be real in the universes of the multiverse, by virtue of the Schrodinger Eq.(20). And therefore, the universes of the multiverse exist.

To ensure the wave functions Eq. (39) of the hydrogen atom in the universes of the multiverse were correct, they were mathematically checked in Subsect. A.

A. A Mathematical Check of the Wave Functions of the Hydrogen Atom of the Universes of the Multiverse:

Only one radial wave function Eq. (30) could be checked at a time. The radial wave function $R_{(k)10}$ ($k=k, n=1, l=0$), corresponding to the ground state of the hydrogen atom, was selected to be checked. Thus, the wave functions of the ground state of the hydrogen atom of the universes of the multiverse $\psi_{(k)10}$ needed to be examined. These wave functions were made up of factors $R_{(k)10}$, Y_0^m, $f_k(t)$.

Checking the radial wave functions $R_{(k)10}$, determined from the radial wave functions Eq. (30), would be lengthy. The spherical harmonics Y_0^m were not checked because they remained unchanged and were not related to the universe quantum number. The functions of time in the universes of the multiverse Eq. (38) were checked once for all quantum numbers.

The ground state radial wave functions $R_{(k)10}$ for the hydrogen atoms were compiled from Eq. (30).

$$R(r)_{(k)10} = \left(\frac{2}{a_k}\right)^{\frac{3}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} e^{-\frac{r}{a_k}} L_0^1\left(\frac{2r}{a_k}\right) = \frac{2}{a_k^{\frac{3}{2}}} e^{-\frac{r}{a_k}} L_0^1\left(\frac{2r}{a_k}\right) \tag{59}$$

And the associated Laguerre polynomial Eq. (31) with n=1 and l=0 of degree zero were

$$L_0^1\left(\frac{2r}{a_k}\right) = \sum_{i=0}^{0}\frac{(-1)^0[1!]^2\left(\frac{2r}{a_k}\right)^0}{0!(0)!(1)!} = 1 \tag{60}$$

Thus, substituting Eq. (60) into the ground state radial wave functions Eq. (59), we had

$$R_{(k)10} = \frac{2e^{-\frac{r}{a_k}}}{a_k^{\frac{3}{2}}} \tag{61}$$

Bohr radii of the universes of the multiverse Eq. (14) were substituted into ground state radial wave functions Eq. (61) to put them in terms of classical physical constants.

$$R_{(k)10} = \frac{2e^{\frac{-r}{\alpha^{-k}\frac{\hbar^2}{m_e e^2}}}}{\left(\alpha^{-k}\frac{\hbar^2}{m_e e^2}\right)^{\frac{3}{2}}} = \frac{2}{\alpha^{-\frac{3}{2}k}\left(\frac{\hbar^2}{m_e e^2}\right)^{\frac{3}{2}}} e^{\frac{-r}{\alpha^{-k}\frac{\hbar^2}{m_e e^2}}} = \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{62}$$

Then, radial wave functions Eq. (62) derived from Eq. (30) were shown to be solutions of the radial type of Eq. (29). This was accomplished in three steps. Radial wave function Eq. (62) were substituted into the left-hand sides of the radial type of Eq. (29) to obtain left hand side expressions. Radial wave functions Eq. (62) were substituted into the right-hand sides of the radial type of Eq. (29) to obtain right hand side expressions. When the two expressions matched the radial wave functions Eq. (62) were solutions.

Eq. (62) were substituted into the left-hand sides of radial type of Eq. (29) that had l equal to zero and the expressions were

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d}{dr}\left[\frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right]\right)\right] - \alpha^{-3k}\frac{e^2}{r}\frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{63}$$

These were simplified. The latter expressions became

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d}{dr}\left[\frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right]\right)\right] - \frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{64}$$

Next the first inner derivatives were evaluated, and we had

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(-\frac{2\alpha^{\frac{5}{2}k} m_e^{\frac{5}{2}} e^5}{\hbar^5} r^2 e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right)\right] - \frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{65}$$

Then the second remaining derivatives were evaluated, and the latter expressions became

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[-\frac{4\alpha^{\frac{5}{2}k} m_e^{\frac{5}{2}} e^5}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} r + \frac{2\alpha^{\frac{7}{2}k} m_e^{\frac{7}{2}} e^7}{\hbar^7} r^2 e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right] - \frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{66}$$

The two terms in the brackets were multiplied by the initial terms and the latter expressions were

$$\frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} - \frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} - \frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \tag{67}$$

Because the first and third terms were equal except for sign, the left-hand sides of the radial type of Eq. (29) in terms of classical physical constants were

$$-\frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \quad (68)$$

Next the right-hand sides of the radial type of Eq. (29) were addressed. When Eq. (62) and Eq. (34) were substituted into the right-hand sides of radial type of Eq. (29) with n equal to one and l equal to zero, these expressions became

$$-\alpha^2 \alpha^{-2k} \frac{m_e c^2}{2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \quad (69)$$

When Eq. (17) squared was substituted for α^2 in Eq. (69) we had

$$-\left(\frac{e^2}{\hbar c}\right)^2 \alpha^{-2k} \frac{m_e c^2}{2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} = -\alpha^{-2k} \frac{m_e e^4}{2\hbar^2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \quad (70)$$

When terms were simplified, the latter expressions became

$$-\frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} \quad (71)$$

Thus, it was seen that the left-hand sides of the radial type of Eq. (29), expressions Eq. (68), matched the right-hand sides of the radial type of Eq. (29), expressions Eq. (71). Therefore, ground state radial wave functions Eq. (62) and the total energy eigenvalues Eq. (34) of the hydrogen atom were correct for principal quantum number one, azimuthal quantum number zero, and any universe quantum number.

The functions of time Eq. (38) were checked as solutions to their source Eq. (36) for all quantum numbers. Eq. (34) and Eq. (38) were substituted into source Eq. (36), and we had

$$\frac{1}{e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}} \frac{d\left[e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k \frac{-\alpha^{2-2k}\frac{m_e c^2}{2n^2}}{\hbar} \quad (72)$$

This is the same as

$$\frac{1}{e^{i(\alpha)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}} \frac{d\left[e^{i(\alpha)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k \frac{-(\alpha)^2\alpha^{-2k}\frac{m_e c^2}{2n^2}}{\hbar} \quad (73)$$

When Eq. (17) was substituted for α in $(\alpha)^2$ in Eq. (73) we had

$$\frac{1}{e^{i\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}} \frac{d\left[e^{i\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k \frac{-\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-2k}\frac{m_e c^2}{2n^2}}{\hbar} \quad (74)$$

This can be simplified to

$$\frac{1}{e^{i\frac{\alpha^{-k} m_e e^4}{2\hbar^3 n^2}t}} \frac{d\left[e^{i\frac{\alpha^{-k} m_e e^4}{2\hbar^3 n^2}t}\right]}{dt} = i\alpha^{-k} \frac{m_e e^4}{2\hbar^3 n^2} \quad (75)$$

Derivatives were taken on the left-hand sides of the latter equations, and we had

$$i\alpha^{-k} \frac{m_e e^4}{2\hbar^3 n^2} = i\alpha^{-k} \frac{m_e e^4}{2\hbar^3 n^2} \quad (76)$$

The left-hand sides of Eq. (36) matched the right-hand sides and hence Eq. (38) checked for all quantum numbers.

It was now shown that ground state wave functions of the hydrogen atom $\psi_{(k)10m}=R_{(k)10}Y_0^m f_k(t)$ were correct for principal quantum number one, azimuthal quantum number zero, any magnetic quantum number, and any universe quantum number. This means the wave functions for the ground state of the hydrogen atom Eq. (39) in the universes of the multiverse were correct and real. Thus, wave functions of atoms of the chemical elements were real in the universes of the multiverse by virtue of the Schrodinger Eq. (20). And it was validated that the universes of the multiverse exist.

5. The Important New Principle of an Atomic Particle Transition Involving the Universes of the Multiverse

An atomic particle transition of the hydrogen atom would be a change of the hydrogen atom, from one discrete energy level to another, along with a change in the discrete mass and size of the electron and proton. An atomic particle transition of the hydrogen atom would produce a hydrogen atom that was a scale-model of itself. It would appear discontinuous as the hydrogen atom "jumps" from the quantum state of one universe of the multiverse to the quantum state of another universe. An atomic particle transition of the hydrogen atom would change the universe quantum number, the universe quantum state, and the universe of the hydrogen atom.

An atomic particle transition and/or an atomic electron transition of the hydrogen atom would result in the emission or absorption of a discrete photon. Because total energy needs to be conserved, the discrete energy of a photon emitted or absorbed would be equal to the difference in total energy eigenvalues Eq. (34). Thus,

$$E_{i/f} = -\alpha^{2-2k_i}\frac{m_e c^2}{2n_i^2} - \left[-\alpha^{2-2k_f}\frac{m_e c^2}{2n_f^2}\right] = \alpha^2 \frac{m_e c^2}{2}\left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2}\right] \quad (77)$$

where the energy of the photon emitted or absorbed from the electron of the hydrogen atom in an atomic particle transition and/or an atomic electron transition was $E_{i/f}$, k_i was the initial universe quantum number of the atomic particle transition of the hydrogen atom, k_f was the final universe quantum number of the atomic particle transition of the hydrogen atom, n_i was the initial principal quantum number of the atomic electron transition of the hydrogen atom, n_f was the final principal quantum number of the atomic electron transition of the hydrogen atom, i/f represents the initial i and final f universe quantum numbers to which something refers in an atomic particle transition and/or an atomic electron transition. If the energy was positive the photon was emitted and conversely if the energy was negative the photon was absorbed.

The atomic particle transition would be an especially important new principle. It was hypothesized that it applied to all atoms. Thus, it would be used to transition the quantum state of a quantum system between the quantum states of consecutive universes of the multiverse. And it would be used with the hydrogen atom to generate a huge force.

TABLE III shows the magnitude of the discrete energy of a photon Eq. (77) that would be required to be emitted or absorbed when the ground state of the hydrogen atom jumped between the universes of the multiverse.

TABLE III. Calculated approximate values of the magnitude of the discrete energy of a photon that would be required to be emitted or absorbed when the ground state of the hydrogen atom jumped between the universes of the multiverse.

Universe k	$E_{i/f}$ *absorbed/emitted*	*Magnitude of* $E_{i/f}$ *(eV)*
0 / 4	$\alpha^{-6}\frac{m_ec^2}{2}$	1.69×10^{18}
0 / 3	$\alpha^{-4}\frac{m_ec^2}{2}$	8.99×10^{13}
0 / 2	$\alpha^{-2}\frac{m_ec^2}{2}$	4.79×10^{9}
0 / 1	$\frac{m_ec^2}{2}$	2.55×10^{5}
0 /−1	$-\alpha^{2}\frac{m_ec^2}{2}$	13.6
−1 /−2	$-\alpha^{4}\frac{m_ec^2}{2}$	7.25×10^{-4}
−2 /−3	$-\alpha^{6}\frac{m_ec^2}{2}$	3.86×10^{-8}
−3/−4	$-\alpha^{8}\frac{m_ec^2}{2}$	2.06×10^{-12}

Note the enormous amount of photon energy emitted when the ground state of the hydrogen atom jumped to a positive universe, as shown in TABLE III. When the ground state of the hydrogen atom jumped from universe zero to universe one the photon energy emitted by the electron was approximately equal to one half the equivalent energy of the rest mass of the electron, $-m_ec^2/2$, as shown in TABLE III. The approximate allowed energy level of the ground state of the hydrogen atom that jumped to universe quantum number one would be the absolute value of, $-m_ec^2/2$, as shown in TABLE II.

When the ground state of the hydrogen atom jumped from the quantum state of universe zero to universe two the photon energy emitted by the electron would be huge. The equivalent energy of the proton would have to be multiplied by a factor of approximately five to equal the photon energy emitted. This photon would be the basis of an extremely energetic powerful reaction engine.

Since the photon absorbed or emitted by the electron of the hydrogen atom that jumped between energy levels requires a definite energy, the photon absorbed or emitted must have a definite wavelength. Wavelength is found from the Planck-Einstein relation $E = h\nu$ and the formula $f = c/\lambda$. That gives us

$$\lambda = c/f = c/\frac{E}{h} = \frac{hc}{E} \tag{78}$$

where frequency is ν or f, wavelength is λ, and the energy of a photon is E. Because E is positive, and $E_{i/f}$ could be negative, it was necessary for E of Eq. (77) to be equal to the absolute value. Substituting Eq. (16) where k was k_i, Eq. (8) where k was k_i, and the absolute value of Eq. (77) into Eq. (78), the wavelength of the photon absorbed or emitted would be

$$\lambda_{i/f} = \frac{h_{k_i}c_{k_i}}{|E_{i/f}|} = \frac{\alpha^{-k_i}h\alpha^{-2k_i}c}{\left|\alpha^2\frac{m_ec^2}{2}\left[\frac{\alpha^{-2k_f}}{n_f^2}-\frac{\alpha^{-2k_i}}{n_i^2}\right]\right|} = \frac{2\alpha^{-2-3k_i}h}{m_ec\left|\left[\frac{\alpha^{-2k_f}}{n_f^2}-\frac{\alpha^{-2k_i}}{n_i^2}\right]\right|} \tag{79}$$

where $\lambda_{i/f}$ was the wavelength of the photon absorbed or emitted when the hydrogen atom jumped between the universes of the multiverse. TABLE IV shows the discrete wavelength Eq. (79) of the photon whose energy would be absorbed or emitted by the ground state of the hydrogen atom when the hydrogen atom jumped between the universes of the multiverse.

TABLE IV. Calculated approximate values of the discrete wavelength of the photon whose energy would be absorbed or emitted by the ground state of the hydrogen atom when the hydrogen atom jumped between the universes of the multiverse.

Universe k	$\lambda_{i/f}$	*Wavelength (m)*
0 / 4	$2\alpha^6h/m_ec$	7.34×10^{-25}
0 / 3	$2\alpha^4h/m_ec$	1.38×10^{-2}
0 / 2	$2\alpha^2h/m_ec$	2.58×10^{-16}
0 / 1	$2h/m_ec$	4.85×10^{-12}
0 /−1	$2\alpha^{-2}h/m_ec$	9.10×10^{-8}
−1 /−2	$2\alpha^{-1}h/m_ec$	6.64×10^{-10}
−2 /−3	$2h/m_ec$	4.85×10^{-12}
−3 /− 4	$2\alpha h/m_ec$	3.54×10^{-14}

The discrete approximate wavelength of the photon absorbed or emitted when the ground state of the hydrogen atom jumped between the quantum state of universe zero and one of 4.85 × 10^{-12} meters (6.19$x10^{19}$ Hz) as shown in TABLE IV, would be an X-ray. The discrete approximate wavelength of the photon absorbed or emitted when the ground state of the hydrogen atom jumped between the quantum state of universe zero and negative one of 9.10 × 10^{-8} meters (3.30$x10^{15}$ Hz) as shown in TABLE IV, would be ultraviolet light.

The discrete approximate wavelength of the photon absorbed or emitted when the ground state of the hydrogen atom jumped between the quantum state of universe zero and two of 2.58 × 10^{-16} meters (1.16$x10^{24}$ Hz) as shown in TABLE IV, would be a gamma ray. These photons emitted by atomic particle transitions when the ground state of the hydrogen atom jumped between the quantum states of universe zero and universe two would be the basis of an extremely powerful reaction engine that emitted gamma rays.

It was hypothesized that atomic particle transitions are possible for all atoms as well as hydrogen. Thus, a future spacecraft could be transitioned between the quantum states of consecutive universes of the multiverse by simultaneous atomic particle transitions. Once it was possible to transition a self contained spacecraft's quantum state to that of a consecutive greater or lesser numbered universe it would automatically be possible to transition the quantum state to the next greater or lesser numbered universe. This happens because what works in one universe's quantum state works the same in another universe's quantum state. This would provide, among other things, the availability of velocity in greater numbered universes of the multiverse described by Eq. (3) and acceleration in numbered universes of the multiverse described by Eq. (82). Thus, there would be no limit on the velocity or acceleration of a future self-contained spacecraft using this concept.

Multiple atomic particle transitions would be the basis of a powerful reaction engine, producing thrust by ejecting photon relativistic mass rearward, in accordance with Newton's third law. The reaction engines thrust could be used to propel a future self-contained spacecraft. Simultaneous multiple atomic particle transitions would transition the future self-contained spacecraft between the quantum states of consecutive universes of the multiverse. Transitioning between the quantum states of the universes of the multiverse would be the only way to reach other universes. This propulsion system would operate on hydrogen fuel. The exhaust would be hydrogen transitioned into the universe k_f. Atomic particle transitions provide energetic economic ecological electromagnetic energy that propagates at the speed of light in a vacuum c_{ki} .

6. Physical Laws of the Universes of the Multiverse

From Eqs. (2), (3) and the fact that velocities are displacements divided by the corresponding elapsed time, the following equations were written for the universes of the multiverse:

$$\Delta t_k = \frac{\Delta x_k}{v_k} = \frac{\alpha^{-k}\Delta x}{\alpha^{-2k}v} = \frac{\alpha^{-k}\Delta x}{\alpha^{-2k}\frac{\Delta x}{\Delta t}} = \alpha^k \Delta t \quad (80)$$

where discrete elapsed time in the universes of the multiverse were Δt_k and the elapsed time in our universe is Δt. This would be interpreted to mean that elapsed time depends on the universe quantum number of the quantum system involved. In other words, the elapsed time, as demonstrated by the time between ticks of a clock would be determined by the universe quantum number of the physical system of the clock. A greater universe quantum number would cause time to hasten, and a lesser universe quantum number would cause time to dilate in a discrete manner. This time change would have to be considered by future self-contained spacecraft that could operate in the quantum state of any universe.

Frequency is one divided by a time (period). Thus, from Eq. (80), discrete frequencies of the universes of the multiverse were

$$f_k = \frac{1}{\Delta t_k} = \frac{1}{\alpha^k \Delta t} = \alpha^{-k}\frac{1}{\Delta t} = \alpha^{-k} f \quad (81)$$

where discrete frequencies of the universes of the multiverse were f_k , frequency in our universe is f, and the time (period) of frequency f is Δt. So, the frequencies of the atoms of a quantum system would determine how much time passes.

Acceleration is the derivative of velocity; thus, substituting Eq. (3) and the equivalent $dt_k = \alpha_k dt$ from Eq. (80) gave

$$a_k = \frac{d}{dt_k}v_k = \frac{d}{\alpha^k dt}\alpha^{-2k}v = \alpha^{-3k}\frac{d}{dt}v = \alpha^{-3k}a \quad (82)$$

where accelerations in the universes of the multiverse were a_k and acceleration in our universe is a. Thus, acceleration of future spacecraft would not be limited in the quantum states of greater numbered universes of the multiverse.

Atomic density is mass per unit volume; thus, from Eqs. (2), (5)

$$\rho_k = \frac{m_k}{V_k} = \frac{\alpha^{2k}m}{\alpha^{-3k}\Delta x^3} = \alpha^{5k}\rho \quad (83)$$

where discrete atomic densities in the universes of the multiverse were ρ_k, atomic density in our universe is ρ, and volumes in the universes of the multiverse were V_k. Thus, the difference in atomic density between consecutive universes of the multiverse would be a factor of approximately 48.3billion (137^5) to one. This change in density would coincide with the intelligently managed disappearance of a quantum system from the quantum state of its present universe and the reappearance in the quantum state of another universe.

Mass energy equivalence is $E = m/c^2$. When mass of particles Eq. (5) and the speeds of light in a vacuum Eq. (8) of the universes of the multiverse were substituted into the universe equations for mass-energy equivalence,

$$E_k = m_k c_k^2 = \alpha^{2k} m(\alpha^{-2k}c)^2 = \alpha^{-2k}mc^2 = \alpha^{-2k}E \quad (84)$$

where mass-energy equivalence of a particle in the universes of the multiverse were E_k, mass-energy equivalence of a particle in our universe is E, masses of particles in the universes of the multiverse were m_k, and the mass of a particle in our universe is m. Thus, a particle of a greater numbered universe would have less mass but more mass-energy equivalence. Particles of a lesser numbered universe would have more mass but less mass-energy equivalence.

7. Electromagnetic Waves and Photons of the Universes of the Multiverse

Discrete ordinary frequencies of electromagnetic waves in the universes of the multiverse were given by Eq. (81). It was hypothesized that the electromagnetic waves with discrete ordinary frequencies f_k were of the same nature as the electromagnetic waves with ordinary frequency f. The "same nature" meant that if an electromagnetic wave with ordinary frequency f exhibited properties like those of a radio wave, light wave, microwave, X-ray, or gamma ray then the electromagnetic waves with discrete ordinary frequencies f_k exhibited the same properties in its own universes of the multiverse. Based on this and the fact that atoms and quantum systems were scale-models of themselves it was hypothesized that the environments of separate universes of the multiverse would be much the same.

Wavelength is equal to the speed of light divided by its frequency of oscillation. When Eqs. (8), (81) were substituted it would be the equation for the universes of the multiverse. Thus,

$$\lambda_k = \frac{c_k}{f_k} = \frac{\alpha^{-2k}c}{\alpha^{-k}f} = \alpha^{-k}\lambda \tag{85}$$

where discrete wavelengths of the electromagnetic wave in the universes of the multiverse were λ_k , and the wavelength of an electromagnetic wave in our universe is λ. It was hypothesized that electromagnetic waves and photons of the universes of the multiverse, related by the fine structure constant as in Eq. (1) to other physical properties besides frequency, would be of the same nature. Thus, the electromagnetic waves of the universes of the multiverse with discrete wavelengths λ_k would be of the same nature as the electromagnetic waves of our universe with wavelengths λ.

The energy-momentum relation is $E^2=(pc)^2+(m_0c^2)^2$. Momentum of photons in the universes of the multiverse, derived from the rearranged energy-momentum relation that had zero rest mass, would be

$$p_k = E_K/c_k = \frac{\alpha^{-2k}E}{\alpha^{-2k}c} = p \tag{86}$$

where, μ for E was negative two as the Gaussian units are the same as Eq. (84), momentum of a photon in the universes of the multiverse were p_k , momentum of a photon in our universe is p, discrete energy of a photon in the universes of the multiverse were E_k, and the energy of a photon is E in our universe. Thus, surprisingly, or not, the momentum of a photon would not depend on which universe it comes from.

It was hypothesized that the relativistic masses of photons m_{relk} in the universes of the multiverse equaled $\alpha^{2k}m_{\mathrm{rel}}$, the same as masses of particles in the universes of the multiverse Eq. (5). Thus,

$$m_{relk} = \alpha^{2k}m_{rel} \tag{87}$$

where discrete relativistic masses of photons in the universes of the multiverse were m_{relk} and the relativistic mass of a photon is m_{rel} in our universe.

And it was hypothesized that the momentum of a photon, using its relativistic mass, was the same as the momentum of a particle; that was equal to its mass multiplied by its velocity. Then, substituting Eq. (8) and Eq. (87) into the equation for the momentum of a photon in the universes of the multiverse gives

$$p_k = m_{relk}c_k = \alpha^{2k}m_{rel}\alpha^{-2k}c = m_{rel}c = p \tag{88}$$

where momentum of a photon in the universes of the multiverse were p_k, and momentum of a photon in our universe is p. This would be a factor in understanding the gravity of the universes of the multiverse. Thus, photons of the same nature would be of the same momentum and vice versa. This equation is superfluous to Eq. (86).

The energy-momentum relation is $E^2 = (pc)^2 + (m_0c^2)^2$. Eqs. (8), (86) were substituted into the energy-momentum relations for the universes of the multiverse that had zero rest mass that gave

$$E_k = p_kc_k = m_{relk}{c_k}^2 = \alpha^{2k}m_{rel}(\alpha^{-2k}c)^2 = \alpha^{-2k}m_{rel}c^2 = \alpha^{-2k}E \tag{89}$$

where discrete energy of photons in the universes of the multiverse were E_k and energy of a photon in our universe is E. The photons of the universes of the multiverse with discrete energies E_k would be of the same nature as a photon of our universe with energy E. This same nature of properties between universes of the multiverse would be another indication of the matching environments of the universes of the multiverse.

Formulas for relativistic masses of photons in the universes of the multiverse, were found by equating $E_k=m_{relk}c_k^2$ Eq. (89) to the Planck-Einstein relation $E = h\upsilon$ in the universes of the multiverse. Thus,

$$m_{relk} = \frac{h_kf_k}{c_k^2} = \frac{\alpha^{-k}h\alpha^{-k}f}{\alpha^{-4k}c^2} = \alpha^{2k}\frac{hf}{c^2} = \alpha^{2k}m_{rel} \tag{90}$$

where frequency is υ, discrete frequencies in the universes of the multiverse were f_k , and frequency in our universe is f. Thus, the relativistic mass of a photon was calculated for the universes of the multiverse. Photons of the universes of the multiverse with discrete relativistic masses m_{relk} would be of the same nature as a photon of our universe with relativistic mass m_{rel}.

It was hypothesized that a powerful photon reaction engine, which runs on hydrogen, could be obtained in the universes of the multiverse by harnessing the energy of photons emitted from the hydrogen atoms during atomic particle transitions.

What follows would be the derivation of the thrust and power of this hydrogen photon reaction engine: This derivation assumed a one hundred percent efficient engine. The maximum thrust for this hydrogen photon reaction engine would be from the ground state of the hydrogen atom. Thus, the principal quantum numbers n_i and n_f would be one.

The absolute value of $E_{i/f}$ Eq. (77) was substituted for E into the rearranged Planck-Einstein relation $E=h\nu$ in the universes of the multiverse where the frequency ν was the frequency $E_{i/f}$. Then the frequency of the emitted or absorbed photon of the hydrogen atom that undergoes an atomic particle transition and/or an atomic electron transition in the universes of the multiverse would be

$$f_{i/f} = \frac{|E_{i/f}|}{h_{k_i}} = \frac{|E_{i/f}|}{\alpha^{-k_i}h} = \alpha^{2+k_i}\frac{m_ec^2}{2h}\left|\left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2}\right]\right| \tag{91}$$

where the frequency of the emitted or absorbed photon of the hydrogen atom that undergoes an atomic particle transition and/or an atomic electron transition in the universes of the multiverse were $f_{i/f}$. To determine the thrust of a hydrogen photon reaction engine the rate of photons emitted in an atomic particle transition must be specified. The frequency of the emitted photons $f_{i/f}$ in an atomic particle transition was selected for this rate. Thus, the electromagnetic wave or thrust would be continuous for one second. For a body, whose mass is constant, Newton's second law of motion can be expressed as $F = ma$. It was hypothesized that this law also applies to relativistic mass. The equation for thrust of a hydrogen photon reaction engine in the universes of the multiverse was obtained from this law. Thus, the thrust of the hydrogen photon reaction engine emitting photons for one second at frequency $f_{i/f}$ would be

$$T_{i/f} = F_{i/f} = M_{relk_i} a_{k_i} = \frac{f_{i/f} m_{relk_i} f_{i/f} D_{k_i}}{(f_{i/f} t_{i/f})^2} = \frac{m_{relk_i}}{t_{i/f}} \frac{D_{k_i}}{t_{i/f}} = \dot{m}_{relk_i} v_{k_i} = \dot{m}_{relk_i} c_{k_i} \tag{92}$$

where $T_{i/f}$ was the thrust for one second caused by propagating photons emitted from the photon reaction engine, at frequency $f_{i/f}$, $F_{i/f}$ was the total force for one second caused by propagating photons emitted from the photon reaction engine, at frequency $f_{i/f}$, M_{relk_i} was the total photon relativistic mass emitted by the reaction engine in one second, a_{k_i} was the hypothetical acceleration of the photons emitted for one second from the reaction engine, m_{relk_i} was the relativistic mass of a single photon being emitted from the reaction engine, D_{k_i} was the distance a single emitted photon would radiate during one rotation of the electron of the hydrogen atom, $t_{i/f}$ was the reciprocal of $f_{i/f}$; equal to the time (period) for one photon to be emitted from the reaction engine, $\dot{m}_{relk_i}$ was the relativistic mass flow rate for one second of photons being emitted from the reaction engine at frequency $f_{i/f}$, and v_{k_i} was the velocity of photons emitted from the reaction engine for one second at frequency $f_{i/f}$.

From $\frac{m_{relk_i}}{t_{i/f}} = \dot{m}_{relk_i}$ Eq. (92) the relativistic mass flow rate of the photons being emitted from the reaction engine in the universes of the multiverse would be

$$\dot{m}_{relk_i} = \frac{m_{relk_i}}{t_{i/f}} = m_{relk_i} f_{i/f} \tag{93}$$

Substituting $m_{relk} = \alpha^{2k} \frac{hf}{c^2}$ Eq. (90), where f equals $f_{i/f}$ and k equals k_i and Eq. (91) into Eq. (93) the relativistic mass flow rate of photons emitted from the reaction engine would be

$$\dot{m}_{relk_i} = \alpha^{2k_i} \frac{hf_{i/f}}{c^2} \alpha^{2+k_i} \frac{m_e c^2}{2h} \left|\left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]\right| \alpha^{-2k_i} = \alpha^{2+3k_i} \frac{f_{i/f} m_e}{2} \left|\left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]\right| \tag{94}$$

Substituting Eq. (91) into Eq. (94) the relativistic mass flow rate $\dot{m}_{relk_i}$ of the photons emitted from the reaction engine was then

$$\dot{m}_{relk_i} = \alpha^{2+3k_i} \frac{\alpha^{2+k_i} \frac{m_e c^2}{2h} \left|\left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]\right| m_e}{2} \left|\left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]\right|$$
$$= \alpha^{4+4k_i} \frac{m_e{}^2 c^2}{4h} \left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]^2. \tag{95}$$

Substituting Eq. (95) and Eq. (8) where k was k_i into $T_{i/f} = \dot{m}_{relk_i} C_{k_i}$ Eq. (92) the hydrogen photon reaction engine thrust would be

$$T_{i/f} = \alpha^{4+4k_i} \frac{m_e{}^2 c^2}{4h} \left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]^2 \alpha^{-2k_i} c = \alpha^{4+2k_i} \frac{m_e{}^2 c^3}{4h} \left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]^2 \tag{96}$$

where $T_{i/f}$ was thrust for one second caused by propagating photons emitted from the hydrogen photon reaction engine, at frequency $f_{i/f}$. The number $f_{i/f}$ would be the number of hydrogen atoms transitioned in one second to obtain the thrust generated for one second. This thrust would be the minimum at frequency $f_{i/f}$ for continuous thrust for one second. Multiplying the thrust $T_{i/f}$ by the actual number of hydrogen atoms transitioned for one second divided by the frequency $f_{i/f}$ determines the actual thrust for one second.

Future spacecraft would be able to use this silent, efficient, energetic, powerful, economic, and ecological reaction engine. This engine thrust would allow travel at any velocity arbitrarily close to the speed of light of any universe.

Power is work divided by elapsed time. Work is force multiplied by distance. Distance divided by elapsed time is velocity. Thrust has the same units as force. Thus, the power of the hydrogen photon reaction engine $P_{i/f}$ was

$$P_{i/f} = \frac{W_{i/f}}{f_{i/f} t_{i/f}} = \frac{F_{i/f} f_{i/f} D_{k_i}}{f_{i/f} t_{i/f}} = F_{i/f} v_{k_i} = T_{i/f} c_{k_i} \tag{97}$$

where $P_{i/f}$ was the power of the hydrogen photon reaction engine radiating photons, at frequency $f_{i/f}$ and $W_{i/f}$ was the total work done by the photon reaction engine radiating photons, at frequency $f_{i/f}$. Then substituting Eq. (96) and Eq. (8) where k was k_i into Eq. (97), the power of a hydrogen photon reaction engine was

$$P_{i/f} = \alpha^{4+2k_i} \frac{m_e{}^2 c^3}{4h} \left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]^2 \alpha^{-2k_i} c = \alpha^{4} \frac{m_e{}^2 c^4}{4h} \left[\alpha^{-2k_f} - \alpha^{-2k_i}\right]^2 \tag{98}$$

The number $f_{i/f}$ would be the number of hydrogen atoms transitioned in one second to obtain the power $P_{i/f}$ generated. Multiplying the power $P_{i/f}$ by the actual number of hydrogen atoms transitioned in one second divided by the frequency $f_{i/f}$ determines the actual power.

This hydrogen photon reaction engine could be used to propel a future self-contained spacecraft. A future self-contained spacecraft would be able to transition between the quantum states of consecutive universes of the multiverse. It would use simultaneous atomic particle transitions to accomplish this. It would be a fact that there would be no limit on the velocity or acceleration of this future self-contained spacecraft in three-dimensional space.

It was hypothesized that the discrete unlimited universes of the multiverse simultaneously exist everywhere in three-dimensional space. It was hypothesized that individually the universes of the multiverse were homogeneous and isotropic when viewed on a large scale. Thus, any lesser numbered universe was always surrounded by greater numbered universes. So, it was hypothesized that gravity in the unlimited universes of the multiverse was caused by the radiation of luminous bodies of greater numbered universes surrounding the particles of lesser numbered universes.

It is known that electromagnetic waves and gravity are governed by the inverse square law.

The velocity of a photon Eq. (8) traveling past particle one and particle two of a greater numbered universe would be $c_k=\alpha^{(-2k)}\ c$.

The velocity of this photon would be the same in our universe as it was in a greater numbered universe. Then the time duration of a greater numbered universe photon to pass by particle one and particle two of our universe, as shown by this equation, would be almost instantaneous.

The momentum of a photon propagating from a greater numbered universe to our universe would be Eq. (88). So $p_k = m_{relk} c_k = p$.

Thus, the momentum of a photon from a greater numbered universe in our universe would be independent of the universe number. Thus, momentum from photons of greater numbered universes would add algebraically to our universe.

It was hypothesized that the density of a photon in a greater numbered universe would be relativistic mass per unit volume, thus using Eqs. (87),

$$\rho_{pk} = \frac{m_{relk}}{V_k} = \frac{\alpha^{2k} m_{rel}}{\alpha^{-3k}\Delta x^3} = \alpha^{5k}\rho_p \qquad (99)$$

where discrete density of a photon in a greater numbered universe would be ρ_{pk} and density of the photon in our universe is ρ_p. The density of a photon in our universe would be the same as the density of this photon in a greater numbered universe.

From Eq. (99) photons of greater numbered universes would be at least a factor of 48.3 billion to one less dense than photons in our universe. It was hypothesized that photons in our universe would be less dense than atoms in our universe. Thus, based on density of a photon Eq. (99) it was hypothesized that photons of a greater numbered universe would pass entirely through atoms of our universe as though the atoms of our universe were not there. In other words, it was hypothesized that the photons of greater numbered universes would transmit through all atoms of our universe as though they were radio waves in air.

Since an isolated photon would have a constant velocity and relativistic mass, it was hypothesized that its force on a particle could be expressed as mass flow rate multiplied by velocity. Thus, using $m_{rel}c = p$ Eq. (88), the force of a photon, of a specific frequency, on a particle of a greater numbered universe k would be

$$F_k = \dot{m}_{relk} c_k = \frac{m_{relk} c_k}{t_k} = \alpha^{-k}\frac{m_{rel}c}{t} = \alpha^{-k}\frac{p}{t} = \alpha^{-k} f p \qquad (100)$$

where f_k was the frequency of a photon of a greater numbered universe k, f is frequency in our universe, F_k was the force of the photon of frequency f_k on a particle of a greater numbered universe k, $\dot{m}_{relk}$ was the photon relativistic mass flow rate to the particle of a greater numbered universe k, $\dot{m}_{relk}$ was the relativistic mass of the photon of a greater numbered universe k, t_k was the time (period) and reciprocal of the frequency f_k of a greater numbered universe k, m_{rel} is the relativistic mass of the photon of frequency f in our universe, t is the time (period) and reciprocal of the frequency f in our universe, and p is the momentum of the photon of frequency f in our universe. This force Eq. (100) put on a particle of our universe would be the same as the force put on a particle of a greater numbered universe k. Thus, the greater the universe number the greater the photon force of a single photon on a particle of our universe.

It was hypothesized that the aggregate photon force on a particle of our universe would decrease progressively with each greater numbered universe. This would be due to the size and geometry of the photon sources. It is hypothesized that this would continue until the photon force was less than that provided by a single photon. From this point on greater numbered universes would provide zero photon force on a particle of our universe.

So, it was hypothesized that the aggregate sum of photons from greater numbered universes would provide a significant finite photon force on every particle of our universe.

And further, it was hypothesized that every particle in our universe would be pushed, by photons from greater numbered universes surrounding them, towards every other particle in our universe, that was directly proportional to the product of the particle masses and inversely proportional to the square of the distance between the particles. The direction of the gravitational force would be external to the particles, the opposite of "action at a distance" imagined by Isaac Newton. The proportionality constant would be the universal gravitational constant.

Thus, the force of gravity in our universe would be

$$F = G\frac{m_1 m_2}{r^2} \qquad (101)$$

where this equation is Isaac Newton's universal law of gravitation, F was a gravitational force in our universe caused by propagating photons from greater numbered universes of the multiverse, G is the universal gravitational constant, m_1 is mass of particle one, m_2 is mass of particle two, and r is the distance between the centers of mass, one and two.

The force of gravity F_K in the universes of the multiverse would be uniform since it was hypothesized that individual universes of the multiverse were homogeneous and isotropic. Then $F_k=\alpha^{\mu k}F$ where μ would be negative one as determined from inspection of $\dot{F}_k=\alpha^{2k}m\alpha^{-3k}a$. Thus, the force of quantum gravity in the universes of the multiverse would be

$$F_K = G_k\frac{m_{1k}m_{2k}}{{r_k}^2} = \alpha^{-k}G\frac{m_1 m_2}{r^2} = \alpha^{-7k}G\frac{\alpha^{2k}m_1\alpha^{2k}m_2}{\left(\alpha^{-k}r\right)^2} \qquad (102)$$

where $\dot{F}_k$ was force in the universes of the multiverse due to Newton's second law of motion, a is acceleration, F_K was the gravitational force in the universe k, G_k were the universal gravitational constant in the universes of the multiverse, μ for m_1 and m_2 was positive two, μ for r was negative one, and

$G_k = \alpha^{-7k}G$ in the universes of the multiverse. Thus, future self-contained spacecraft would have to consider F_K, the gravitational force in the universe k.

In the future it would be known that gravity was due to the force of photons Eq. (100) from greater numbered universes than the universe in which gravity was measured. Thus, artificial anti-g-forces would be possible. Artificially generated greater universe numbered photons would provide this force. Future self-contained spacecraft would undergo extreme environmental conditions due to inertial and acceleration g-forces. However, it would be possible to obtain continuous normal gravity (1g) in the interior of operating self-contained spacecraft using this artificial anti-g-force.

8. Consequences of Einstein's Special Theory of Relativity in the Universes of the Multiverse

Lorentz factors in the universes of the multiverse were found by substituting instantaneous velocities Eq. (3) and speeds of light in a vacuum Eq. (8) into the formula for the Lorentz factor $\gamma=1/\sqrt{1-v^2/c^2}$.

$$\gamma_k = \frac{1}{\sqrt{1-v_k^2/c_k^2}} = \frac{1}{\sqrt{1-(\alpha^{-2k}v)^2/(\alpha^{-2k}c)^2}} = \frac{1}{\sqrt{1-v^2/c^2}} = \gamma \qquad (103)$$

where Lorentz factors in the universes of the multiverse were γ_k and the Lorentz factor in our universe is γ. Thus, the Lorentz factor would be a dimensionless physical factor that would not depend on the universe's number. This means that when a moving quantum system underwent a change in its universe number or multiple simultaneous atomic particle transitions, its dimensionless Lorentz factor would remain the same.

The equation for relativistic mass-energy equivalence of a particle in our universe was hypothesized to be,

$$E = m_{rel}c^2 = \gamma mc^2 = \frac{1}{\sqrt{1-v^2/c^2}}mc^2 \qquad (104)$$

where E is the relativistic equivalent energy of a particle in our universe, m_{rel} is the relativistic mass of a particle in our universe, and m is the mass of a particle in our universe. Substituting Eqs. (5), (8), (103) into Eq. (104), relativistic mass-energy equivalence in the universes of the multiverse would be

$$E_K = \gamma_k m_k c_k^2 = \gamma\alpha^{2k}m(\alpha^{-2k}c)^2 = \alpha^{-2k}\gamma mc^2 = \alpha^{-2k}\frac{1}{\sqrt{1-{v_k}^2/{c_k}^2}}mc^2 \qquad (105)$$

where E_K were discrete relativistic equivalent energies of a particle in the universes of the multiverse. Thus, the velocity of a particle in a universe would be limited to below the speed of light in a vacuum for its universe number.

Relativistic time dilation indicates that, for an observer in an inertial frame of reference, a clock that is moving relative to them will measure a difference in elapsed time less than a clock that is at rest in its frame of reference. When elapsed time of the universes of the multiverse Eq. (80) were inserted for a classical elapsed time the concept of relativistic time dilation $\Delta t' = \gamma\Delta t$ was written in the universes of the multiverse as

$$\Delta t_k' = \gamma\Delta t_k = \alpha^k\gamma\Delta t \qquad (106)$$

where relativistic elapsed time in the universes of the multiverse, for quantum systems traveling at nonzero velocities in the universes of the multiverse, were $\Delta t_k'$. Relativistic time in the universes of the multiverse would be dilated or hastened.

Relativistic frequency is one divided by the relativistic time (period). Thus, from Eq. (106), the relativistic frequencies of the universes of the multiverse were

$$f_k' = \frac{1}{\Delta t_k'} = \alpha^{-k}\frac{1}{\gamma\Delta t} = \alpha^{-k}\frac{f}{\gamma} \qquad (107)$$

where f_k' were relativistic frequencies in the universes of the multiverse. Thus, time would be a property of a quantum system, where a quantum system relativistic or normal time was related to the electron orbital frequencies of the atoms of the quantum system.

Relativistic length contraction is the phenomenon where the length of a moving object undergoes a contraction along the dimension of motion as seen from a stationary reference frame. When displacements in the universes of the multiverse Eq. (2) were substituted for the classical displacement Δx the concept of relativistic length contraction $\Delta x' = \Delta x/\gamma$ was written in the universes of the multiverse as

$$\Delta x_k' = \frac{\Delta x_k}{\gamma} = \alpha^{-k}\frac{\Delta x}{\gamma} \qquad (108)$$

where relativistic lengths along the dimension of motion in the universes of the multiverse would be $\Delta x_k'$. It was hypothesized that $\Delta x_k'$ applies to all displacements. Thus, all atoms, quantum systems, electromagnetic waves, and photons of the universes of the multiverse would remain scale-models of themselves under normal and relativistic conditions.

Then, when α^{-k}/γ in Eq. (108) equaled one there would be no change in the size of a quantum system, from the size at rest in our universe, when a quantum system traveled at this discrete speed beyond the speed of light in a vacuum c. And, if α^{-k}/γ in Eq. (108) equaled one, then $\alpha^k\gamma$ in Eq. (106) would equal one, and there would be no change in the size or time of the quantum system, from the size or time at rest in our universe, when a quantum system traveled at this discrete speed beyond the speed of light in a vacuum c. The expression α^{-k}/γ is approximately equal to $137^k/\gamma$. Thus, the subject discrete speeds when α^{-k}/γ equaled one must be extremely close to the universe's speed of light in a vacuum. However, this discrete velocity allows travel in the quantum state of the subject universes with no relativistic effects.

Author Declaration Section

- Conflict of Interest: The author has no conflicts to disclose.
- Author contributions: Richard C. (Cal) Havens is the sole author

9. Conclusion

The purpose of this work was to investigate the existence of new physical constants. Their existence is now a fact. These physical constants define the universes of the multiverse. The only difference between the universes of the multiverse is the quantum states of the atoms. Quantum states, physical systems, electromagnetic waves, photons, etc. in the universes of the multiverse are scale-models of themselves. This makes the universes of the multiverse much the same.

This work provided the first plausible theory for a multiverse. The limitless universes of the multiverse comprise everything that exists; the entirety of space, matter, energy, time, gravity, and the physical constants and laws that describe them.

It is concluded that there could be particles in the vacuum of three-dimensional space, which could travel faster than the speed of light in a vacuum c, which do not violate the theory of special relativity. However, the velocity of a particle is limited to below the speed of light in a vacuum c_k of its universe. Discrete speeds of light in a vacuum increase with greater numbered universes and decrease with lesser numbered universes. Also, it is concluded that it is possible to travel in the quantum state of some universes with no relativistic effects when $\alpha^k\gamma$ is equal to one.

It is concluded that Schrodinger equations can solve the quantum systems of the hydrogen atom of the universes of the multiverse. Discrete wave functions and discrete properties of the hydrogen atom in the universes of the multiverse, are analytically derived. And the wave functions of the hydrogen atom of the universes of the multiverse are shown to be valid, by a mathematical check of the ground state of the hydrogen atom. This validates the existence of the universes of the multiverse as the Schrodinger equations apply to all atoms in all universes.

The hydrogen atom is transitioned between the universes of the multiverse by an atomic particle transition. After the transition, the hydrogen atom is a scale-model of itself. In this process a photon is emitted or absorbed. The energy level, frequency, and wavelength of the emitted or absorbed photon are derived. Thus, it is for all atoms.

The equations that relate the wavelength of an electromagnetic wave to its frequency in the universes of the multiverse are determined.

The thrust and power of a hydrogen photon reaction engine is determined that operates on the new principle of atomic particle transitions. Transitioning between the quantum states of consecutive universes of the multiverse, a future self-contained spacecraft would have no limit on velocity or acceleration in three-dimensional space. The future self-contained spacecraft would use extreme artificial anti-g-forces to maintain the interior of the operating self-contained spacecraft at normal gravity (1g). Transitioning between the quantum states of consecutive universes of the multiverse a future self-contained spacecraft would disappear from the quantum state of its initial universe and reappear in the quantum state of its final universe.

Quantum gravity of the universes of the multiverse is caused by photons from luminous bodies propagating from greater numbered universes of the multiverse to lesser numbered universes of the multiverse. This reinstates Isaac Newton's universal law of gravitation. The universal law of gravitation is determined for the universes of the multiverse.

It is concluded that time is a property of a quantum system, where relativistic or normal time of a quantum system is related to the electron orbital frequencies of the atoms of the quantum system. Time passes fast in the less dense and less massive quantum states of greater numbered universes of the multiverse. Time passes slowly in the denser more massive quantum states of lesser numbered universes of the multiverse.

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