ISSN: 2641-8835

Madridge Journal of Bioinformatics and Systems Biology

Research Article Open Access

Common fixed point theorem in intuitionistic Fuzzy metric Spaces using compatible mappings of type (A)

Saurabh Manro*

School of Mathematics and Computer Applications, Thapar University, Patiala, India

Article Info

***Corresponding author:**
Saurabh Manro
School of Mathematics and Computer Applications
Thapar University
Patiala, India
E-mail: sauravmanro@hotmail.com

Received: December 5, 2018
Accepted: December 11, 2018
Published: December 17, 2018

Citation: Manro S. Common fixed point theorem in intuitionistic Fuzzy metric Spaces using compatible mappings of type (A). *Madridge J Bioinform Syst Biol.* 2018; 1(1): 5-9.
doi: 10.18689/mjbsb-1000102

Abstract

In this paper, we prove common fixed point theorem in intuitionistic fuzzy metric space using compatible mappings of type (A).

Keywords: Intuitionistic Fuzzy metric space; Compatible mappings of type (A); Common fixed point.

AMS (2010) Subject Classification: 47H10, 54H25

Introduction

Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [5] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al.[1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [3]. In this paper, we prove common fixed point theorem in intuitionistic fuzzy metric space using compatible mappings of type (A).

Preliminaries

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [4] in study of statistical metric spaces.

Definition [6] A binary operation $*$: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0,1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition [6] A binary operation $\Diamond$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if $\Diamond$ satisfies the following conditions:

(i) $\Diamond$ is commutative and associative;

(ii) $\Diamond$ is continuous;

(iii) $a \Diamond 0 = a$ for all $a \in [0, 1]$;

(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for all *a*, *b*, *c*, *d* ∈[0,1] .

Alaca et al. [1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [3] as:

Definition [1] A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) M(x, y, t) + N(x, y, t) ≤ 1 for all $x, y \in X$ and t > 0;

(ii) M(x, y, 0) = 0 for all $x, y \in X$;

(iii) M(x, y, t) = 1 for all $x, y \in X$; and t > 0 if and only if x = y;

(iv) M(x, y, t) = M(y, x, t) for all $x, y \in X$ and t > 0;

(v) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) for all $x, y, z \in X$ and s, t > 0;

(vi) for all $x, y \in X$, M(x, y, .) : [0, ∞) → [0, 1] is left continuous;

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and t > 0;

(viii) N(x, y, 0) = 1 for all $x, y \in X$;

(ix) N(x, y, t) = 0 for all $x, y \in X$ and t > 0 if and only if x = y;

(x) N(x, y, t) = N(y, x, t) for all $x, y \in X$ and t > 0;

(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all $x, y, z \in X$ and s, t > 0;

(xii) for all x, y∈X, N(x, y, .) : [0, ∞)→[0, 1] is right continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1: Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as x ◊ y = 1-((1-x) * (1-y)) for all $x, y \in X$.

Remark 2.2: In intuitionistic fuzzy metric space (X, M, N, *, ◊), M (x, y, *) is non-decreasing and N(x, y, ◊) is non-increasing for all $x, y \in X$.

Alaca, Turkoglu and Yildiz [1] introduced the following notions:

Definition Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x∈X if, for all t > 0, $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Definition [1] an intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 2.1: Let X = {1/n: $n \in N$} ∪ {0} and let * be the continuous t-norm and ◊ be the continuous t-conorm defined by a * b = ab and a ◊ b = min{1, a+b} respectively, for all *a*, *b* ∈ [0,1]. For each $t \in (0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \dfrac{t}{t+|x-y|}, & t>0, \\ 0 & t=0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|}, & t>0 \\ 1 & t=0 \end{cases}$$

Clearly, (X, M, N, *, ◊) is complete intuitionistic fuzzy metric space.

Definition A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for all t > 0, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some u in X.

Definition A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be compatible of type (A) if$\lim_{n\to\infty} M(fgx_n, ggx_n, t) = 1$, $\lim_{n\to\infty} N(fgx_n, ggx_n, t) = 0$ and $\lim_{n\to\infty} M(gfx_n, ffx_n, t) = 1$, $\lim_{n\to\infty} N(gfx_n, ffx_n, t) = 0$.

for all t > 0, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some u in X.

Alaca [1] proved the following results:

Lemma Let (X, M, N, *, ◊) be intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if for a number *k*>1 such that

M(x, y, kt) ≤ M(x, y, t) and N(x, y, kt) ≥ N(x, y, t) Then x = y.

Lemma Let (X, M, N, *, ◊) be intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if for a number *k* > 1 such that

$M(y_{n+2}, y_{n+1}, t) \geq M(y_{n+1}, y_n, kt)$, $N(y_{n+2}, y_{n+1}, t) \leq N(y_{n+1}, y_n, kt)$.

Then $\{y_n\}$ is a Cauchy sequence in X.

Lemma Let f and g be compatible self mappings of type (A) of a complete intuitionistic fuzzy metric space (*X, M, N*,*, ◊) with *a* **b* = min{*a*, *b*} and *a*◊*b* = max{*a*, *b*} for all *a*, *b* ∈ [0,1] and fu = gu for some $u \in X$. Then gfu = fgu = ffu = ggu.

Results

Theorem: Let (*X, M, N*,*, ◊) be a complete intuitionistic fuzzy metric space with *a* **b* = min{*a*, *b*} and *a*◊*b* = max{*a*, *b*} for all *a*, *b* ∈[0, 1]. Let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(3.1) $P(X) \subseteq ST(X)$, $Q(X) \subseteq AB(X)$,

(3.2) AB = BA, ST = TS, PB = BP, QT = TQ,

(3.3) P or AB is continuous,

(3.4) (P, AB) and (Q, ST) are pairs of compatible mappings of type (A),

(3.5) there exist $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

M (*Px*,*Qy*, *kt*) ≥ *M* (*ABx*, *STy*,*t*) * *M* (*Px*, *ABx*, *t*) * *M* (*Qy*, *STy*, *t*) * *M* (*Px*, *STy*, *t*)

N (*Px*,*Qy*, *kt*) ≤ *N* (*ABx*, *STy*, *t*)◊*N* (*Px*, *ABx*, *t*) ◊*N* (*Qy*, *STy*, *t*)◊*N*

(Px, STy, t)

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof

Forexistence:

Let $x_0 \in X_2$, from (3.1), there exist $x_1, x_2 \in X$ such that

$Px_0 = STx_1, Qx_1 = ABx_2$. Inductively, we Construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Px_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = y_{2n}$

for n = 1, 2, 3, …

Take $x = x_{2n,}\ y = x_{2n}+_1$ in (3.5), we get

$M(Px_{2n}, Qx_{2n+1}, kt) \ge M(ABx_{2n}, STx_{2n+1}, t) * M(Px_{2n}, ABx_{2n}, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Px_{2n}, STx_{2n+1}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \ge M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n,} t) * M(y_{2n+2,} y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \ge M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)$

$M(y_{2n+1}, y_{2n+2}, kt) \ge M(y_{2n}, y_{2n+1}, t)$

And

$N(Px_{2n}, Qx_{2n+1}, kt) \le N(ABx_{2n}, STx_{2n+1}, t) \Diamond N(Px_{2n}, ABx_{2n}, t) \Diamond N(Qx_{2n+1}, STx_{2n+1}, t) \Diamond N(Px_{2n}, STx_{2n+1}, t)$

$N(y_{2n+1}, y_{2n+2}, kt) \le N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n}, t) \Diamond N(y_{2n+2}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+2,} t)$

$N(y_{2n+1}, y_{2n+2}, kt) \le N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+2}, t)$

$N(y_{2n+1}, y_{2n+2}, kt) \le N(y_{2n}, y_{2n+1}, t)$

Similarly, $M(y_{2n+2}, y_{2n+3}, kt) \ge M(y_{2n+1}, y_{2n+2}, t)$, $N(y_{2n+2}, y_{2n+3}, kt) \le N(y_{2n+1}, y_{2n+2}, t)$.

Thus, we have

$M(y_{n+1}, y_{n+2}, kt) \ge M(y_n, y_{n+1}, t)$, $N(y_{n+1}, y_{n+2}, kt) \le N(y_n, y_{n+1}, t)$ for n = 1,2,3,….

Therefore, we have

$M(y_n, y_{n+1}, t) \ge M(y_n, y_{n+1}, t/q) \ge M(y_{n-1}, y_n, t/q^2) \ge \ldots \ge M(y_1, y_2, t/q^n) \to 1$,

$N(y_n, y_{n+1}, t) \le N(y_n, y_{n+1}, t/q) \le N(y_{n-1}, y_n, t/q^2) \le \ldots \le N(y_1, y_2, t/q^n) \to 0$

when $n \to \infty$.

For each $\in > 0$ and $t > 0$, we can choose $n_0 \in$ N such that

$M(y_n, y_{n+1}, t) > 1 - \in$, $N(y_n, y_{n+1}, t) < \in$ for all $n \ge n_0$.

For $m, n \in \mathbb{N}$, we suppose $m \ge n$. Then, we have

$M(y_n, y_m, t) \ge M(y_n, y_{n+1}, t/m-n) * M(y_{n+1}, y_{n+2}, t/m-n) * \ldots * M(y_{m-1}, y_m, t/m-n)$

$> ((1-\in) * (1-\in) * \ldots * (1-\in))(m-n)$ *times*

$\ge (1-\in)$

$N(y_n, y_m, t) \le N(y_n, y_{n+1}, t/m-n) \Diamond N(y_{n+1}, y_{n+2}, t/m-n) \Diamond \ldots \Diamond N(y_{m-1}, y_m, t/m-n)$

$< ((\in) \Diamond (\in) \Diamond \ldots \Diamond (\in))(m-n)$ *times*

$\le (\in)$ $M(y_n, y_m, t) > (1-\in)$, $N(y_n, y_m, t) < \in$

and hence $\{y_n\}$ is a Cauchy sequence in X. As X is complete, $\{y_n\}$ converges to some point

$z \in X$. Also, its subsequences converge to this point $z \in X$.

i.e. $\{Qx2_{n+1}\} \to z, \{STx_{2n+1}\} \to z, \{Px_{2n}\} \to z, \{ABx_{2n}\} \to z$.

Suppose AB is continuous.

As AB is continuous, we have $\{ABABx_{2n}\} \to ABz, \{ABPx_{2n}\} \to ABz$.

As (P, AB) is compatible pair of type (A), we have $\{PABx_{2n}\} \to ABz$.

Take $x = ABx_{2n}, y = x_{2n+1}$ in (3.5), we get

$M(PABx_{2n}, Qx_{2n+1}, kt) \ge M(ABABx_{2n}, STx_{2n+1}, t) * M(PABx_{2n}, ABABx_{2n}, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(PABx_{2n}, STx_{2n+1}, t)$

$n \to \infty$

$M(ABz, z, kt) \ge M(ABz, z, t) * M(ABz, ABz, t) * M(z, z, t) *$

$M(ABz, z, t)$ $M(ABz, z, kt) \ge M(ABz, z, t)$

And

$N(PABx_{2n}, Qx_{2n+1}, kt) \le N(ABABx_{2n}, STx_{2n+1}, t) \Diamond N(PABx_{2n}, ABABx_{2n}, t) \Diamond N(Qx_{2n+1}, STx_{2n+1}, t) \Diamond N(PABx_{2n}, STx_{2n+1}, t)$

$n \to \infty$

$N(ABz, z, kt) \le N(ABz, z, t) \Diamond N(ABz, ABz, t) \Diamond N(z, z, t) \Diamond$

$N(ABz, z, t)$ $N(ABz, z, kt) \le N(ABz, z, t)$

By lemma, ABz = z.

Next, we show that Pz = z.

Put x = z and $y = x_{2n}$ in (3.5), we get

$M(Pz, Qx_{2n}, kt) \ge M(ABz, STx_{2n}, t) * M(Pz, ABz, t) * M(Qx_{2n}, STx_{2n,} t) * M(Pz, STx_{2n}, t)$

$n \to \infty$

$M(Pz, z, kt) \ge M(z, z, t) * M(Pz, z, t) * M(z, z, t) * M(Pz, z, t)$ $M(Pz, z, kt) \ge M(Pz, z, t)$

And

$N(Pz, Qx_{2n}, kt) \le N(ABz, STx_{2n}, t) \Diamond N(Pz, ABz, t) \Diamond N(Qx_{2n}, STx_{2n}, t) \Diamond N(Pz, STx_{2n}, t)$

$n \to \infty$

$N(Pz, z, kt) \le N(z, z, t) \Diamond N(Pz, z, t) \Diamond N(z, z, t) \Diamond N(Pz, z, t)$

$N(Pz, z, kt) \le N(Pz, z, t)$

Therefore, ABz = z = Pz.

Now, we show that Bz =z.

Put x = Bz and $y = x_{2n-1}$ in (3.5), we get

$M(PBz, Qx_{2n-1}, kt) \ge M(ABBz, STx_{2n-1}, t) * M(PBz, ABBz, t) * M$

$(Qx_{2n-1}, STx_{2n-1}, t) * M (PBz, STx_{2n-1}, t)$

$N (PBz, Qx_{2n-1}, kt) \le N (ABBz, STx_{2n-1}, t) \lozenge N (PBz, ABBz, t) \lozenge N (Qx_{2n-1}, STx_{2n-1}, t) \lozenge N (PBz, STx_{2n-1}, t)$

As BP = PB and AB = BA, so that

P(Bz) = (PB)z = BPz = Bz and (AB)(Bz) = (BA)(Bz) = B(AB)z = Bz.

Taking, $n \to \infty$, we get

$M (Bz, z, kt) \ge M (Bz, z, t) * M (Bz, Bz, t) * M (z, z, t) * M (Bz, z, t)$

$M (Bz, z, kt) \ge M (Bz, z, t)$

And

$N (Bz, z, kt) \le N (Bz, z, t) \lozenge N (Bz, Bz, t) \lozenge N (z, z, t) \lozenge N (Bz, z, t)$

$N (Bz, z, kt) \le N (Bz, z, t)$.

Therefore, by using lemma, we get Bz = z and also we have, ABz = z. Therefore, Az = Bz = Pz = z.

As $P(X) \subseteq ST (X)$, there exist $u \in X$ such that $z = Pz = STu$.

Putting, $x = x_{2n,}$ $y = u$ in (3.5), we get

$M (Px_{2n}, Qu, kt) \ge M (ABx_{2n}, STu, t) * M (Px_{2n}, ABx_{2n}, t) * M (Qu, STu, t) * M (Px_{2n}, STu, t)$

$n \to \infty$

$M (z, Qu, kt) \ge M (z, z, t) * M (z, z, t) * M (Qu, z, t) * M (z, z, t)$

$M (z, Qu, kt) \ge M (Qu, z, t)$

And

$N (Px_{2n}, Qu, kt) \le N (ABx_{2n}, STu, t) \lozenge N (Px_{2n}, ABx_{2n}, t) \lozenge N (Qu, STu, t) \lozenge N (Px_{2n}, STu, t)$

$n \to \infty$ N

$(z, Qu, kt) \le N (z, z, t) \lozenge N (z, z, t) \lozenge N (Qu, z, t) \lozenge N (z, z, t)$

$N (z, Qu, kt) \le N (z, Qu, t)$

By using lemma, we get Qu = z. Hence, STu = z = Qu.

Since (Q, ST) is compatible pair of type (A), therefore, by lemma, we have QSTu = STQu. Therefore, Qz = STz.

Now, we show that Qz = z.

Take $x = x_{2n,}$ $y = z$ in (3.5), we get

$M (Px_{2n}, Qz, kt) \ge M (ABx_{2n}, STz, t) * M (Px_{2n}, ABx_{2n}, t) * M (Qz, STz, t) * M (Px_{2n}, STz, t)$

$n \to \infty$

$M (z, Qz, kt) \ge M (z, Qz, t) * M (z, z, t) * M (Qz, Qz, t) * M (z, Qz, t)$

$M (z, Qz, kt) \ge M (z, Qz, t)$

and

$N (Px_{2n}, Qz, kt) \le N (ABx_{2n}, STz, t) \lozenge N (Px_{2n}, ABx_{2n}, t) \lozenge N (Qz, STz, t) \lozenge N (Px_{2n}, STz, t)$

$n \to \infty$

$N (z, Qz, kt) \le N (z, Qz, t) \lozenge N (z, z, t) \lozenge N (Qz, Qz, t) \lozenge N (z, Qz, t)$

$N (z, Qz, kt) \le N (z, Qz, t)$

Therefore, by using lemma, Qz = z.

As QT = TQ, ST = TS, we have QTz = TQz = Tz and STTz = TSTz = TQz = Tz.

Next, we claim that Tz = z.

For this, take $= x_{2n,}$ $y = Tz$ in (3.5), we get

$M (Px_{2n}, QTz, kt) \ge M (ABx_{2n}, STTz, t) * M (Px_{2n}, ABx_{2n}, t) * M (Qz, STTz, t) * M (Px_{2n}, STTz, t)$

$n \to \infty$

$M (z,Tz, kt) \ge M (z,Tz, t) * M (z, z, t) * M (Tz,Tz, t) * M (z,Tz, t)$

$M (z,Tz, kt) \ge M (z,Tz, t)$

and

$N (Px_{2n}, QTz, kt) \le N (ABx_{2n}, STTz, t) \lozenge N (Px_{2n}, ABx_{2n}, t) \lozenge N (QTz, STTz, t) \lozenge N (Px_{2n}, STTz, t)$

$n \to \infty$

$N (z,Tz, kt) \le N (z,Tz, t) \lozenge N (z, z, t) \lozenge N (Tz,Tz, t) \lozenge N (z,Tz, t)$

$N (z,Tz, kt) \le N (z,Tz, t)$

therefore, by lemma, we get Tz = z. as STz = Qz = z = Tz. This gives, Sz =z. Hence, Az = Bz = Pz = Qz = Sz = Tz = z. Hence, z is a common fixed point of A, B, S, T, P and Q. The proof is similar P is continuous.

For uniqueness

Let u is another fixed point of A, B, S, T, P and Q. Therefore, take x = z and y = u in (3.5), we get

$M (Pz, Qu, kt) \ge M (ABz, STu, t) * M (Pz, ABz, t) * M (Qu, STu, t) * M (Pz, STu, t)$

$M (z, u, kt) \ge M (z, u, t) * M (z, z, t) * M (u, u, t) * M (z, u, t)$

$M (z, u, kt) \ge M (z, u, t)$

And $N (Pz, Qu, kt) \le N (ABz, STu, t) \lozenge N (Pz, ABz, t) \lozenge N (Qu, STu, t) \lozenge N (Pz, STu, t)$

$N (z, u, kt) \le N (z, u, t) \lozenge N (z, z, t) \lozenge N (u, u, t) \lozenge N (z, u, t)$

$N (z,u, kt) \le N (z, u, t)$

By lemma, we get z = u. Hence, z is a unique common fixed point of A, B, S, T, P and Q. Take B = T = I (Identity map), then theorem 3.1 becomes:

Corollary 3.1: Let $(X, M, N, *, \lozenge)$ be a complete intuitionistic fuzzy metric space with $a * b = \min\{a, b\}$ and $a \lozenge b = \max\{a, b\}$ for all $a, b \in [0,1]$. Let A, S, P and Q be mappings from X in to itself such that the following conditions are satisfied:

(3.6) $P(X) \subseteq S(X)$, $Q(X) \subseteq A(X)$,

(3.7) P or A is continuous,

(3.8) (P, A) and (Q, S) are pairs of compatible mappings of type (A), (3.9) there exist $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$M(Px, Qy, kt) \geq M(Ax, Sy, t) * M(Px, Ax, t) * M(Qy, Sy, t) * M(Px, Sy, t)$

$N(Px, Qy, kt) \leq N(Ax, Sy, t) \Diamond N(Px, Ax, t) \Diamond N(Qy, Sy, t) \Diamond N(Px, Sy, t)$

Then A, S, P and Q have a unique common fixed point in X.

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